Structural Competency for Architects
Answer Key

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1-1  Find the resultant vector magnitude and direction for the forces shown in the diagram.

\[ Ax = 28k(\cos30) = 24.25\text{←} = -24.25 \]
\[ Bx = (26k/13)(5) = 10\text{→} = 10.00 \]
\[ Cx = (25k/5)(4) = 20k \text{→} = 20.00 \]
\[ \Sigma fx = -24.25 + 10 + 20 = 5.75 \]
\[ Ay = 28k(\sin30) = 14.00k \uparrow = 14.00 \]
\[ By = (26k/13)(12) = 24.00 \uparrow = 24.00 \]
\[ Cy = (25k/5)(3) = 15k \downarrow = -15.00 \]
\[ \Sigma fy = 14 + 24 - 15 = 23.00 \]

Resultant Force = \( \sqrt{5.75^2 + 23^2} \) = 23.71k

Direction: \( \tan^{-1}(23/5.75) \) = 75.96° above X axis

1-2  Find the resultant vector magnitude and direction for the forces shown in the diagram.

\[ Ax = 0 = 0 \]
\[ Bx = 4.5\text{←} = -4.50 \]
\[ Cx = (5/5)(3)\text{←} = -3.00 \]
\[ Dx = 0 = 0 \]
\[ Ex = (3.75/5)(3)\text{→} = 2.25 \]
\[ Fx = 5.25 \text{→} = 5.25 \]
\[ \Sigma fx = -4.5 - 3 + 2.25 + 5.25 = 0 \]

\[ Ay = A\downarrow = -A \]
\[ By = 0 = 0 \]
\[ Cy = (5/5)(4)\uparrow = 4.00 \]
\[ Dy = 3\uparrow = 3.00 \]
\[ Ey = (3.75/5)(4)\uparrow = 3.00 \]
\[ Fy = 0 = 0 \]
\[ \Sigma fy = -A + 4 + 3 + 3 = 10 - A \]

Find: Force A if resultant = 0

If the truss is static (it doesn’t move), then the sum of all forces must be zero and this means

\[ 10 - A = 0 \quad \text{...} \quad A = 10k \]
1-3 Find the resultant vector magnitude and direction for the forces shown in the diagram.

\[ \sqrt{8^2 + 12^2} = 14.42 \]

\[ 2400 \times \frac{8}{14.42} = 1331.48 \text{# down} \]
\[ 2400 \times \frac{12}{14.42} = 1997.23 \text{# left} \]

\[ \sum f_y = -800 - 1331.48 = -2131.48 \text{#} \]

\[ \sum f_x = -1997.23 \text{#} \]

\[ F = \sqrt{(-2131.48)^2 + (-1997.23)^2} = 2920.98 \text{#} \]

\[ \arctan \left( \frac{-2131.48}{-1997.23} \right) = 46.86^{\circ} \text{ below the - X axis} \]

1-4 Find the moment about point A:

\[ \sum M_A = 2k(0) + 4k(8') + 6k(16') - 3k(4') - 5k(12') + 1k(3') \]
\[ = 0 + 32 + 96 - 12 - 60 + 3 = 59 \text{k-ft} \]

1-5 Find the moment about support A.

\[ 4k(\sin 30) = 2k \downarrow \quad 4k(\cos 30) = 3.46k \rightarrow \]
\[ 2'(\tan 30) = 1.15' \]

\[ \sum M_A = 3.46k (4') - 2k(1.15') + 6(1.15') = 18.44 \text{k-ft} \]
1-6  Find the moment

a) about point A
\[ M_A = 18.46\#(48" - 36") - 7.69\#(15") \]
\[ = 106.17\# - in \]

b) about point B
Find \( M_B \):
\[ M_B = -18.46\#(36") - 7.69\#(15") \]
\[ = -779.91 \# - in \]

1-7  Find Reactions:

Unknows: \( Ax, Ay \) and \( By \)

\[ \sum M_A = 0 = 6k(2') + 8k(10') - By(16') \]
\[ By = \frac{(12+80)}{16} = 5.75k \]

\[ \sum f_y = 0 = Ay - 6k - 8k + 5.75k \]
\[ Ay = 6 + 8 - 5.75 = 8.25k \]

\[ \sum f_x = 0 = Ax \]

ANSWER: \( Ax = 0 \), \( Ay = 8.25k \uparrow \),
\( By = 5.75k \uparrow \)

1-8  Find Reactions:

Unknowns = \( Ay, Bx, By \)

\[ \sum M_B = 0 = -2k(16') + Ay(10') - 12k(8') \]
\[ Ay = \frac{(32+96)}{10} = 12.8k \]

\[ \sum f_y = 0 = -2k + 12.8k - 12k + By \]
\[ By = 2 - 12.8 + 12 = 1.2k \]

\[ \sum f_x = 0 = Bx \]
1-9  Find the reactions:

Unknowns: Ax, Ay and By

\[ \sum M_A = 0 = 12k(3') + (2k/f)(10')(3'+3'+5') - By(16') \]

By = \((36+220)/16\) = 16k

\[ \sum f_y = 0 = Ay - 12k - 2k/f(10') + 16k \]

Ay = 12 + 20 – 16 = 16k

\[ \sum f_x = 0 = Ax \]

ANSWER: Ax = 0, Ay = 16k↑, By = 16k↑

1-10  Find the reactions:

Unknowns: Ax, Ay and By

\[ \sum M_A = 0 = [(3k/f)(5')][5'+5/2'] - By(8') \]

By = \(127.5/8\) = 15.94k

\[ \sum f_y = 0 = Ay - (3k/f)(5') + 15.94k \]

Ay = 15 – 15.94 = -0.94k = 0.94↓

\[ \sum f_x = 0 = Ax \]

ANSWER: Ax = 0, Ay = 0.94↓, By = 15.94k↑

1-11  Find the reactions:

Unknowns: Ax, Ay and By

\[ \sum M_A = 0 = [(3k/f)(6')/2][4'+2'] - By(8') \]

By = \(54/8\) = 6.75k

\[ \sum f_y = 0 = Ay - [(3k/f)(6')/2]k + 6.75k \]

Ay = 9 – 6.75 = 2.25k

\[ \sum f_x = 0 = Ax \]

ANSWER: Ax = 0, Ay = 2.25k↑, By = 6.75k↑
1-12 Find the reactions:

Unknows: Ax, Ay and By

\[ \sum M_A = 0 = \left[ \frac{3k}{f} (15') / 2 \right] (5') + \frac{1}{f} (15') (7.5') - By (15') \]
\[ By = \frac{225}{15} = 15k \]

\[ \sum f_y = 0 = Ay - \left[ \frac{3k}{f} (15') / 2 \right] - \frac{1}{f} (15') + 15k \]
\[ Ay = 22.5 + 15 - 15 = 22.5k \]

\[ \sum f_x = 0 = Ax \]

ANSWER: Ax = 0, Ay = 22.5k↑, By = 15k↑

1-13 Find the reactions:

Unknows: Ax, Ay and By

\[ 30k (4/5) = 24k↓, 30k (3/5) = 18k \rightarrow \]

\[ \sum M_A = 0 = 24k (8') + 18k (6') - 10k (12') - By (16') \]
\[ By = \frac{180}{16} = 11.25↑ \]

\[ \sum f_y = 0 = Ay - 24k + 11.25k \]
\[ Ay = 24 - 11.25 = 12.75k \]

\[ \sum f_x = 0 = Ax - 10k \quad \ldots \quad Ax = 10k \]

ANSWER: Ax = 10k →, Ay = 12.75k↑, By = 11.25k↑

1-14 Find Reactions: Unknows = Ax, Ay, By

\[ \sum M_A = 0 = 6k (2') - 3k (2') - 3k (2') - By (16') \]
\[ By = (12 - 6 - 6) / 16 = 0 \]

\[ \sum f_y = 0 = Ay - 6k + 0 \quad Ay = 6k \]

\[ \sum f_x = 0 = Ax - 3k + 3k \quad Ax = 0 \]
2-1 Solve for bar forces using Method of Joints.

BAR FORCES:
\[
\begin{align*}
AB &= 15.5 \text{k C} \\
BD &= 7.125 \text{k C} \\
DF &= FH = 9 \text{k C} \\
HK &= 6.375 \text{k C} \\
AC &= GJ = 0 \\
CE &= 7.125 \text{T} \\
EG &= 6.375 \text{T} \\
BC &= \sqrt{7.125^2 + 9.5^2} = 11.875 \text{k T} \\
DE &= \sqrt{1.875^2 + 2.5^2} = 3.125 \text{k T} \\
EH &= \sqrt{2.625^2 + 3.5^2} = 4.375 \text{k T} \\
GK &= \sqrt{6.375^2 + 8.5^2} = 10.625 \text{k T}
\end{align*}
\]

2-2 Solve for bar forces using Method of Joints.

JOINT B: 2 equations, 2 unknowns

\[\Sigma_fy = 0 = 10.25 - 9 + BDy + BEy = 1.25 + BD(\sin 30) + BE(\sin 30) = 1.25 + .5BD + .5BE\]

\[BDy = 11.5(sin30) = 11.5(.5) = 5.75k\]

\[\Sigma_fx = 0 = 17.753 + BDx - BEx = 17.753 + BD(cos 30) - BE(cos 30) = 17.753 + .866BD - .866BE\]

\[BDx = 11.5(cos30) = 11.5(.866) = 9.959k\]
2-2 continued

2-3 Solve for bar forces using Method of Joints.

Break 6k into components: \( Y = 6 \left( \frac{2}{\sqrt{5}} \right) = 5.37k \), \( X = 6 \left( \frac{1}{\sqrt{5}} \right) = 2.68k \)

\[ AB = \sqrt{17.75^2 + 10.25^2} = 20.5 \text{ C} \]
\[ BD = DF = \sqrt{(9.959^2 + 5.75^2)} = 11.5 \text{ C} \]
\[ BE = \sqrt{7.794^2 + 4.5^2} = 9.0 \text{ C} \]
\[ EF = \sqrt{5.196^2 + 3.0^2} = 6.0 \text{ C} \]
\[ FH = \sqrt{15.155^2 + 8.75^2} = 17.5 \text{ C} \]

\[ AC = CE = 15.75 \text{ T} \]
\[ BC = FG = 0 \]
\[ DE = 7.5 \text{ T} \]
\[ EG = GH = 15.16 \text{ T} \]

\[ AB = \sqrt{4.03^2 + 2.015^2} = 4.51 \text{ C} \]
\[ AF = EF = 4.03 \text{ T} \]
\[ BC = 0 \]
\[ BD = 4 \text{ T} \]
\[ BE = \sqrt{(10.71^2 + 5.355^2)} = 5.355 \text{ V5} = 11.97 \text{ C} \]
\[ BF = 2 \text{ T} \]
\[ CD = DE = 6 \text{ C} \]

\[ Ay = 2.015 \]
\[ Ey = 11.355 \]
2-4 Find bar force BE and BC using Method of sections

Reactions:
\[ \sum M_D = 0 = 9(7.15) + 13(6) - F_y(18) \]
\[ F_y = 7.91 \]
\[ \sum f_y = 0 = -13 + D_y + 7.91 \quad D_y = 5.09 \]
\[ \sum f_x = 0 = 9 + D_x \quad D_x = -9 = 9 ← \]

Look at right side:
\[ \sum f_y = 0 = 7.91 + B E_y \quad ... \]
\[ B E_y = -7.91 = 7.91 ↓ \]
\[ B E = 7.91 / \sin 50 = 10.33 k \text{ C} \]
\[ \sum M_E = 0 = -7.91 k(6) + B C(6' \tan 50) = -47.46 + 7.15 B C \]
\[ B C = 6.64 \text{ C} \]

2-5 Find bar force DE and DF using Method of sections

Look at top
\[ \sum M_D = 0 = -12k(8') + C E(10') \quad ... \quad C E = 9.6 ↑ \]

Four unknowns: \( D E_x, D E_y, D F_x, D F_y \).
assume \( D E \) and \( D F \) are in compression
\[ D E_x = D E[10 / \sqrt{(10^2 + 8^2)}] = 0.781 D E \]
\[ D E_y = D E[8 / \sqrt{(10^2 + 8^2)}] = 0.625 D E \]
\[ D F_x = F E[4 / \sqrt{(4^2 + 8^2)}] = 0.447 D F \]
\[ D F_y = F E[8 / \sqrt{(4^2 + 8^2)}] = 0.894 D F \]
\[ \sum f_x = 0 = -12 - 10 + .781 D E - .447 D F \]
\[ D E = 28.169 + 0.572 D F \]
\[ \sum f_y = 0 = 9.6 + 0.625 D E + 0.894 D F \]
\[ = 9.6 + 17.606 + .358 F E + 0.894 D F \]
\[ = 27.206 + 1.252 D F \]
\[ D F = -21.73 = 21.73 T \]
\[ D E = 28.169 + 0.572(-21.73) = 15.739 \text{ C} \]
2-6 Find bar force CE, CD, BD using Method of sections

Look at left
\[ \Sigma M_C = 0 = -6k(4') - BD(2')BD = -12 \quad BD = 12 \ C \]

assume CE is in tension, \( CE_x = CE_y \)
\[ \Sigma M_D = 0 = -6k(6') - 4k(2') + CE_x(2') + CE_y(2') = -44 + 4CE_x \]
\( CE_x = CE_y = 11 \quad CE = \sqrt{(11^2 + 11^2)} = 15.56 \)
assume CD is in tension
\[ \Sigma f_y = 0 = -6 - 4 + 11 - CD_y \]
\( CD_y = 1 \downarrow \quad CD = 1.414 \ T \)

2-7 Find bar force in the active tension counters.

\[ \Sigma M_A = 0 = 2(0) + 7(4) + 9(10) + 2(14) - Gy(14) \]
\( Gy = (28 + 90 + 28)/14 = 10.42k \)

\[ \Sigma f_y = 0 = Ay - 2 - 7 - 9 - 2 + 10.42 \]
\( Ay = 2+7+9+2-10.42 = 9.58 \)

Section 1: left side
\[ \Sigma f_y = 0 = 9.58 - 2 + Ty \]
\( Ty = -9.58 + 2 = -7.58k = 7.58 \downarrow \quad BC \text{ is active} \)
\( BC = 7.58[\sqrt{(8^2 + 4^2)}/8] = 8.47k \)

Section 2: left side
\[ \Sigma f_y = 0 = 9.58 - 2 - 7 + Ty \quad Ty = -9.58 + 2 + 7 = 0.58 \downarrow \quad DE \text{ is active} \)
\( DE = 0.58[\sqrt{(8^2 + 6^2)}/8] = 0.73k \)

Section 3: left side
\[ \Sigma f_y = 0 = 9.58 - 2 - 7 - 9 + Ty \quad Ty = -9.58 + 2 + 7 + 9 = 8.42k = 8.42 \uparrow \]
\( EH \text{ is active} \)
\( DE = 8.42[\sqrt{(8^2 + 4^2)}/8] = 9.41k \)
2-8 Find bar force in the active tension counters.

Section 1: top
\[ \sum f_x = 0 = -5 + T_x \]
\[ T_x = 5k = 5 \rightarrow \quad AD \text{ is active} \]
\[ AD = 5\sqrt{(8^2 + 16^2)}/16 = 5.58k \]

Section 2: top
\[ \sum f_x = 0 = -5 + 10 + T_x \quad ... \]
\[ T_y = 5 - 10 = -5 = 5 \leftarrow \quad DE \text{ is active} \]
\[ DE = 5\sqrt{(10^2 + 16^2)}/16 = 5.90k \]

Section 3: top
\[ \sum f_x = 0 = -5 + 10 - 10 + T_x \quad ... \]
\[ T_y = 5 - 10 + 10 = 5 = 5 \rightarrow \quad EH \text{ is active} \]
\[ EH = 5\sqrt{(12^2 + 16^2)}/16 = 6.25k \]
3-1
a) Find the reactions and sag at point C \( h_C \) if the sag at point B, \( h_B = 3' \).

\[ \sum M_A = 0 = 2k(12') + 3k(22') - Dy(32') \]
\[ Dy = 2.8125k \]

\[ \sum F_y = 0 = Ay - 2 - 3 + 2.8125 \]
\[ Ay = 5 - 2.8125 = 2.1875k = Aby \]

\[ ABx/ABy = 12'/3' \]
\[ ABx = 2.1875(12/3) = 8.75k = CDx \]
\[ CDx/CDy = 10/h_c = 8.75/2.8125h_c = 10(2.8125)/8.75 = 3.21' \]

b) Find the sag \( h \) and the reactions at the supports if the maximum tension in leg CD is 8k.

\[ \sum M_A = 0 = 2k(12') + 3k(22') - Dy(32') \]
\[ Dy = 2.8125k \]

\[ Dy^2 + Dx^2 = 8^2 \]
\[ Dx = \sqrt{64 - 2.8125^2} = 7.489k \]
\[ h_C = 10'[2.1825/7.489] = 3.76' \]

3-2 Find the tension in each leg of the cable.

\[ \sum M_A = 0 = 4k(16') + 5k(32') + 2k(46') - Ey(60') \]
\[ Ey = 5.267k \]

\[ \sum F_y = 0 = Ay - 4 - 5 - 2 + 5.267 \]
\[ Ay = 5.733k \]

\[ Ax = 5.733(16'/4') = 22.932k \]

At joint B:
\[ \sum F_y = 0 = 5.733 - 4 + BCy \]
\[ BCy = -1.733k \]

At joint C:
\[ \sum F_y = 0 = 1.733 - 5 + CDy \]
\[ CDy = 3.267k \]

\[ AB = \sqrt{(22.932^2 + 5.733^2)} = 23.638k \]
\[ BC = \sqrt{(22.932^2 + 1.733^2)} = 22.997k \]
\[ CD = \sqrt{(22.932^2 + 3.267^2)} = 23.164k \]
\[ DE = \sqrt{(22.932^2 + 5.267^2)} = 23.529k \]

OR:

Cut section through C, consider left side:

\[ \sum M_C = 0 = 5.73(32) - (4)(16) - 22.92(h_c) \]
\[ h_c = 5.21' \]

Cut section through D, consider right side:

\[ \sum M_C = 0 = -5.267(14) + 22.932(h_d) \]
\[ h_d = 3.22' \]
3-3 Find the reactions and the force in the pin.

Whole arch:
\[ \sum M_A = 0 = 18k(10') + 9K(6') - By(40') \]
By = \( (180 + 54)/40 \) = 5.85k

\[ \sum F_y = 0 = Ay - 18 + 5.85k \]
\[ \sum F_x = 0 = Ax + 9 - Bx \]

\[ \sum M_C = 0 = -18k(10') + 12.15(20') - Ax(9') \]
Ax = 7k and Bx = 7 + 9 = 16k

3-4 Find the reactions and the force in the pin.

SPLIT THE ARCH AT THE PIN.
Left Side:
\[ \sum M_C = 0 = -5k(4') + Ay(13') - Ax(12') \]
Ax = \( (13Ay - 20)/12 \)

\[ \sum M_A = 0 = 18k(10') + 9K(6') - By(40') \]
By = \( (180 + 54)/40 \) = 5.85k

\[ \sum F_y = 0 = Cy - 18 + 12.15 \]
\[ \sum F_x = 0 = Cx + 7 \]

\[ C = \sqrt{5.85^2 + 7^2} = 9.12k \]
3-5 Find the reactions and the force in the pin.

\[ \sum M_A = 0 = 200k(1') - By(8') \]
By = 25

\[ \sum F_y = 0 = Ay - 200 + By \]
Ay = 200 - 25 = 175k

\[ \sum F_x = 0 = Ax - Bx \]
Bx = Ax

SPLIT THE FRAME AT THE PIN.
Left Side:

\[ \sum M_C = 0 = 175k(4') - Ax(4') \]
Ax = 175k

\[ \sum F_x = 0 = Ax - Cx \]
Cx = 175k

\[ \sum F_y = 0 = Cy - Ay \ldots Ay = Cy = 175k \]

\[ C = \sqrt{175^2 + 175^2} = 247.49k \]

3-6 Find the reactions and the force in the pin.

\[ \sum M_A = 0 = 20k(1') + 30k(3') - By(6') \]
By = 110/6 = 18.33k

\[ \sum F_y = 0 = Ay - 20 - 30 + 18.33k \]
Ay = 50 - 18.33 = 31.67k

\[ \sum F_x = 0 = Ax \]

Bar DE:

\[ \sum M_D = 0 = 20k(2') + 30k(4') - Ey(6') \ldots Ey = 26.67k \]

\[ \sum F_y = 0 = Dy - 20 - 30 + 26.67k \ldots Dy = 23.33k \]

Bar DB:

\[ \sum F_y = 0 = Cy + 18.33 + 23.33 \]
Cy = -41.67k

\[ \sum M_D = 0 = -18.33k(5') + 41.67k(3') + Cx(6') \]
Cx = -5.56k

\[ C = \sqrt{5.56^2 + 41.67^2} = 42.04k \]
3-7 Find the reactions and pin forces. Note this frame has three legs:
EAC is one piece, ED is one piece, vertical containing DCB is one piece

\[ \sum M_B = 0 = -2k(2') - 4k(6') + Ay(4') \]
\[ Ay = 7k \sum F_y = 0 = 7k - 2k + By \]
\[ By = -5k = 5k \downarrow \sum F_x = 0 = -4k + Bx \ldots Bx = 4k \]

BAR ED:
\[ \sum M_E = 0 = 2k(2') - Dy(4') \]
\[ Dy = 1k \sum F_y = 0 = Ey - 2 + 1Ey = 1k \]

BAR EAC:
\[ \sum F_y = 0 = -1k + 7k + Cy \ldots Cy = -6k \]
\[ \sum M_E = 0 = 6k(4') - Cx(2') \ldots Cx = 12k \ldots Ex = Dx = 12k \]

Pin Forces: \( E = D = \sqrt{12 + 122} = 12.04k \)
\( C = \sqrt{62 + 122} = 13.42k \)
4-9

\[
M_x = 42x - 12(x-5)^2/2 + 12(x-13)^2/2
\]

\[
V_{\text{max}} = \text{largest reaction} = 54k
\]

\[
M_{\text{max}} \text{ occurs at } V = 0 = \text{ at } x = 5 + 42/12 = 8.5'
\]

\[
M_{\text{max}} = 42(8.5) - 12(3.5^2/2) = 283.5k-f
\]

M = 0 at x = 0, 16'

4-10

\[
M_x = 42x - 12(x-5)^2/2 + 12(x-13)^2/2
\]

\[
V_{\text{max}} = \text{largest reaction} = 54k
\]

\[
M_{\text{max}} \text{ occurs at } V = 0 = \text{ at } x = 5 + 42/12 = 8.5'
\]

\[
M_{\text{max}} = 42(8.5) - 12(3.5^2/2) = 283.5k-f
\]

M = 0 at x = 0, 16'

4-11

\[
W_x = -7 + 7x/12
\]

\[
V_x = 42 - 7x + 7x^2/24
\]

\[
M_x = -168 + 42x - 7x^2/2 + 7x^3/72
\]

\[
V_{\text{max}} = \text{largest reaction} = 16k
\]

\[
M_{\text{max}} \text{ occurs at } V = 0 = \text{ at } x = 3'
\]

\[
M_{\text{max}} = 16(3) = 48k-f
\]

M = 0 at x = 0, 12'

4-12

\[
M_x = 16x - 24(x-3) + 7(x-6) - 1(x-6)^2/2
\]

\[
V_{\text{max}} = \text{largest reaction} = -1k
\]

\[
M_{\text{max}} \text{ occurs at } V = 0 = \text{ at } x = 3'
\]

\[
M_{\text{max}} = 16(3) = 48k-f
\]

M = 0 at x = 0, 12'
5-1 Find the loads on the columns given a uniform floor load of 80psf using tributary area.

\[
\begin{align*}
A1 &= 8'(12')(80) = 7,680#
A2 &= (8'+12')(12')(80) = 19,200#
A3 &= (12')(12')(80') = 11,520#
A4 &= 0
\end{align*}
\]

\[
\begin{align*}
B1 &= (12'+8')(8')(80') = 12,800#
B2 &= [(12'+8')(8') + 12'(12')](80') = 24,320#
B3 &= [(8')(16') + 12'(12')](80') = 21,760#
B4 &= 16'(8')(80psf) = 10,240#
\end{align*}
\]

\[
\begin{align*}
C1 &= 8'(8')(80psf) = 5,120#
C2 &= [8'(8')+ 16'(12')](80psf) = 20,480#
C3 &= [(8')(16') + 16'(12'+16')](80psf) = 46,080#
C4 &= 8'(16')(80psf) = 10,240#
\end{align*}
\]

\[
\begin{align*}
D1 &= 0
D2 &= 16'(12')(80psf) = 15,360#
D3 &= 16'(12'+16')(80psf) = 35,840#
D4 &= 16'(16')(80psf) = 20,480#
\end{align*}
\]

5-2 Find the loads on the columns given a uniform floor load of 80psf using tributary area.

\[
\begin{align*}
A1 &= 20'(20')(80psf) = 32,000#
A2 &= 20'(20')(80psf) = 32,000#
A3 &= 0
\end{align*}
\]

\[
\begin{align*}
B1 &= [16'(16') + 4'(20') + 4'(16')(80') = 32,000#
B2 &= [(20')(4') + 20'(16')+ 16'(16'+16')](80') = 72,960#
B3 &= 16'(16'+16')(80') = 40,960#
\end{align*}
\]

\[
\begin{align*}
C1 &= 4'(16')(80psf) = 5,120#
C2 &= 16'(20' + 16')(80psf) = 46,080#
C3 &= 16'(16')(80psf) = 20,480#
\end{align*}
\]

Total = 281,600#
5-2 Find the loads on the columns given a uniform floor load of 80psf
b) by calculating beam reactions

Bm 1: \( R_1 = R_2 = 80 \text{psf}(4')(40'/2) = 6,400\)#
Bm 2: \( R_1 = R_2 = 80 \text{psf}(8')(40'/2) = 12,800\)#
Bm 3: \( R_1 = R_2 = 80 \text{psf}(8')(16'/2) = 5,120\)#
Bm 4: \( R_1 = R_2 = 5,120#/2 = 2,560\)#
Bm 5: \( R_1 = 80 \text{psf}(4')(40'/2) + (2,560)(16/40) + 80 \text{psf}(4')(16')(8'/40') = 8,448\)#
Bm 6: \( R_1 = R_2 = 80 \text{psf}(8')(24'/2) = 7,680\)#
Bm 7: \( R_1 = R_2 = 7,680#/3(2) = 11,520\)#
Bm 8: \( R_1 = (2,560 + 11,520)(16/40) + 80 \text{psf}(4')(40'/2) = 12,032\)#
\( R_2 = (2,560 + 11,520)(24/40) + 80 \text{psf}(4')(40'/2) = 14,848\)#
Bm 9: \( R_1 = 80\text{psf}(4')(24')(28/40) + 11,520\#(16/40) = 9,984\# \)
\( R_2 = 80\text{psf}(4')(24')(12/40) + 11,520\#(24/40) = 9,216\# \)
Bm 10: \( R_1 = 12,800(32/40) + 12,800(24/40) + 12,032(16/40) + 5,120(8/40) = 23,756.8 \)
\( R_2 = 12,800(8/40) + 12,800(16/40) + 12,032(24/40) + 5,120(32/40) = 18,995.2 \)
Bm 11: \( R_1 = R_2 = 80\text{psf}(4')(32'/2) = 5,120\# \)
Bm 12: \( R_1 = R_2 = 80\text{psf}(8')(32'/2) = 10,240\# \)
Bm 13: \( R_1 = 12,800(32/40) + 12,800(24/40) + (8,448 + 5,120)(16/40) + 10,240(8/40) = 25,395.2\# \)
\( R_2 = 12,800(8/40) + 12,800(16/40) + (8,448 + 5,120)(24/40) + 10,240(32/40) = 24,012.8\# \)
Bm 14: \( R_1 = R_2 = (7,680 + 10,240)(3/2) = 26,880\# \)
Bm 15: \( R_1 = 5,120\#(16/40) + 10,240\#(8/40) = 4,096\# \)
\( R_2 = 5,120\#(24/40) + 10,240\#(32/40) = 11,264\# \)
Bm 16: \( R_1 = R_2 = 10,240\#(3/2) = 15,360\# \)

A1 = 23,756.8 + 6,400 = 30,156.8\#
A2 = 25,395.2 + 6,400 = 31,795.2\#
A3 = 4,096\#

B1 = 18,995.2 + 14,848 = 33,843.2\#
B2 = 12,032 + 24,012.8 + 26,880 + 10,240 = 73,164.8
B3 = 11,264 + 10,240 + 15,360 = 36,864\#

C1 = 9,216\#
C2 = 9,984 + 26,880 + 5,120 = 41,984\#
C3 = 15,360 + 5,120 = 20,480\#

Total = 281,600#

5-3 A uniform wind load of 30psf is resisted at each level by columns A, B and C in figure 5-2. Determine the wind load on each column at each level if levels are 12' o.c.

A: 30psf(12')(20') = 7,200\#
B: 30psf(12')(20' + 16') = 12,960\#
C: 30psf(12')(16') = 5,760\#
6-1. A diagonal tension brace, 15' long and having a round cross-section with a diameter of 3/4" is subjected to 10k of tension. What is the change in length of the brace if $E = 29,000\text{ksi}$?

$$A = \pi \left(\frac{3}{4}\right)^2/4 = 0.4418\text{in}^2$$

$$dL = \frac{PL}{EA} = \frac{10\text{k}(15')(12''/f)/[29,000\text{ksi}(0.4418)]}{0.14''}$$

6-2. A W14X22 with an area, $A = 6.49\text{in}^2$ and a length of 24' is installed on the roof of a building when the temperature is 80°F. What will be the change in length when the temperature drops to 15°F if the coefficient of thermal expansion for steel is $6.5\times10^{-6} \text{in/in/F}$?

$$dL = \alpha L(\Delta T) = (6.5\times10^{-6} \text{in/in/F})(24')(12''/f)(80 - 15\text{^oF}) = 0.12''$$

6-3. What is the required length of the Bronze post if the beam must remain level?

$$P_B = 12\text{k/f}(4')(2'/3') = 32\text{k}$$

$$P_S = 12\text{k/f}(4') - 32\text{k} = 16\text{k}$$

$$\frac{P_SL_S/E_S A_S}{P_BL_B/E_B A_B} = \frac{16\text{k}(36'')/[29000\text{ksi}(1\text{in}^2)]}{32\text{k}(L_B)/[12,000\text{ksi}(2.25\text{in}^2)]}$$

$$L_B = 16\text{k}(36'')(12,000\text{ksi})(2.25\text{in}^2)/[29000\text{ksi}(1\text{in}^2)(32)] = 16.76''$$
6-4. A 12’ canopy supports a load of 600#/f with a hinge at the wall and a cable at the end. The 15’ cable is attached to the wall at some distance h above the canopy. Determine the distance h so that the canopy remains level given the cable properties of: \( E = 29,000 \text{ksi}, A = 1 \text{in}^2 \)

\[
T_y = \frac{600#/f(12’)}{2} = 3600#
\]

\[
T = 3600# \times \frac{(15’)}{h} = 54,000/h#
\]

\[
dL = \frac{(54,000/h)(15')(12\text{in}/f)}{[29,000,000\text{psi}(1\text{in}^2)]} = 0.335”
\]

\[
h^2 + 144”^2 = (15(12\text{in}/f)+.335”)^2
\]

\[
h = 108.557”
\]
7-1: Find $I_x$ and $I_y$ for the cross-sections shown:

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>y</th>
<th>Ay</th>
<th>$I_x$</th>
<th>dy</th>
<th>Ady²</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.00</td>
<td>0.75</td>
<td>4.50</td>
<td>1.13</td>
<td>1.71</td>
<td>17.63</td>
</tr>
<tr>
<td>B</td>
<td>12.00</td>
<td>3.00</td>
<td>36.00</td>
<td>36.00</td>
<td>-0.54</td>
<td>3.44</td>
</tr>
<tr>
<td>C</td>
<td>3.00</td>
<td>3.75</td>
<td>11.25</td>
<td>0.56</td>
<td>-1.29</td>
<td>4.96</td>
</tr>
</tbody>
</table>

sums: 21.00 51.75 37.69 26.04

$Y$: 2.46

$I_x$: 63.73

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>x</th>
<th>Ax</th>
<th>$I_y$</th>
<th>dx</th>
<th>Adx²</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEFT</td>
<td>6.00</td>
<td>2.00</td>
<td>12.00</td>
<td>8.00</td>
<td>1.00</td>
<td>6.00</td>
</tr>
<tr>
<td>RIGHT</td>
<td>3.00</td>
<td>5.00</td>
<td>15.00</td>
<td>1.00</td>
<td>-2.00</td>
<td>12.00</td>
</tr>
</tbody>
</table>

sums: 21.00 63.00 13.00 18.00

$X$: 3.00

$I_y$: 31.00
7-2: Find $I_x$ and $I_y$ for the cross-sections shown:

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>$y$</th>
<th>$Ay$</th>
<th>$Ix$</th>
<th>$dy$</th>
<th>$Ady^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>4.00</td>
<td>4.00</td>
<td>0.08</td>
<td>-1.25</td>
<td>1.56</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>4.00</td>
<td>4.00</td>
<td>0.08</td>
<td>-1.25</td>
<td>1.56</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.08</td>
<td>1.25</td>
<td>1.56</td>
</tr>
<tr>
<td>D</td>
<td>1.00</td>
<td>1.50</td>
<td>1.50</td>
<td>0.08</td>
<td>1.25</td>
<td>1.56</td>
</tr>
<tr>
<td>E</td>
<td>7.00</td>
<td>5.00</td>
<td>35.00</td>
<td>0.58</td>
<td>-2.25</td>
<td>35.44</td>
</tr>
<tr>
<td>F</td>
<td>7.00</td>
<td>0.50</td>
<td>3.50</td>
<td>0.58</td>
<td>2.25</td>
<td>35.44</td>
</tr>
<tr>
<td>G</td>
<td>5.50</td>
<td>2.75</td>
<td>15.13</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>$x$</th>
<th>$Ax$</th>
<th>$ly$</th>
<th>$dx$</th>
<th>$Adx^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.08</td>
<td>3.00</td>
<td>9.00</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>6.50</td>
<td>6.50</td>
<td>0.08</td>
<td>-3.00</td>
<td>9.00</td>
</tr>
<tr>
<td>C</td>
<td>1.00</td>
<td>0.50</td>
<td>0.50</td>
<td>0.08</td>
<td>3.00</td>
<td>9.00</td>
</tr>
<tr>
<td>D</td>
<td>1.00</td>
<td>6.50</td>
<td>6.50</td>
<td>0.08</td>
<td>-3.00</td>
<td>9.00</td>
</tr>
<tr>
<td>E</td>
<td>7.00</td>
<td>3.50</td>
<td>24.50</td>
<td>28.58</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>F</td>
<td>7.00</td>
<td>3.50</td>
<td>24.50</td>
<td>28.58</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>G</td>
<td>5.50</td>
<td>3.50</td>
<td>19.25</td>
<td>0.29</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

sums: 23.50 64.63 15.36 77.13

$y$ 2.75
$lx$ 92.48

sums: 23.50 82.25 57.79 36.00

$x$ 3.50
$ly$ 93.79
7-3: Find $I_x$ and $I_y$ for the cross-sections shown:

![Cross-section diagram](image)

<table>
<thead>
<tr>
<th>Component</th>
<th>$A$</th>
<th>$y$</th>
<th>$Ay$</th>
<th>$I_x$</th>
<th>$dy$</th>
<th>$Ady^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRIANGLE</td>
<td>27.00</td>
<td>3.00</td>
<td>81.00</td>
<td>121.50</td>
<td>0.13</td>
<td>0.47</td>
</tr>
<tr>
<td>CIRCLE</td>
<td>-3.14</td>
<td>2.00</td>
<td>-6.28</td>
<td>-0.79</td>
<td>1.13</td>
<td>-4.02</td>
</tr>
<tr>
<td>sums:</td>
<td>23.86</td>
<td>74.72</td>
<td>120.71</td>
<td></td>
<td>3.56</td>
<td></td>
</tr>
</tbody>
</table>

$Y = 3.13$

$I_x = 117.15$

<table>
<thead>
<tr>
<th>Component</th>
<th>$A$</th>
<th>$x$</th>
<th>$Ax$</th>
<th>$I_y$</th>
<th>$dx$</th>
<th>$Adx^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRIANGLE</td>
<td>27.00</td>
<td>2.00</td>
<td>54.00</td>
<td>54.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>CIRCLE</td>
<td>-3.14</td>
<td>2.00</td>
<td>-6.28</td>
<td>-0.79</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>sums:</td>
<td>23.86</td>
<td>47.72</td>
<td>53.21</td>
<td></td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

$X = 2.00$

$I_y = 53.21$
7-4: Find $I_x$ and $I_y$ for the cross-sections shown:

![Cross-section diagram](image)

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>$y$</th>
<th>$A_y$</th>
<th>$I_x$</th>
<th>$dy$</th>
<th>$Ady^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W8X10</td>
<td>11.70</td>
<td>6.59</td>
<td>77.10</td>
<td>146.00</td>
<td>-1.23</td>
<td>17.70</td>
</tr>
<tr>
<td>C8X13.75</td>
<td>4.04</td>
<td>1.79</td>
<td>7.23</td>
<td>1.52</td>
<td>3.57</td>
<td>51.49</td>
</tr>
</tbody>
</table>

**Sums:**
- $Y = 5.36$
- $I_x = 216.71$

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>$x$</th>
<th>$A_x$</th>
<th>$I_y$</th>
<th>$dx$</th>
<th>$Adx^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W8X10</td>
<td>11.70</td>
<td>0.00</td>
<td>0.00</td>
<td>54.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>C8X13.75</td>
<td>4.04</td>
<td>0.00</td>
<td>0.00</td>
<td>36.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Sums:**
- $X = 0.00$
- $I_y = 90.10$
7-5: Find $I_x$ and $I_y$ for the cross-sections shown:

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Component} & A & y & Ay & I_x & dy & Ady^2 \\
\hline
\text{LEFT} & 3.61 & 3.63 & 13.10 & 8.76 & 0.00 & 0.00 \\
\text{RIGHT} & 3.61 & 3.63 & 13.10 & 8.76 & 0.00 & 0.00 \\
\hline
\text{sums:} & 7.22 & 26.21 & 17.52 & & 0.00 & \\
\hline
Y & 3.63 & & & & & \\
I_x & 17.52 & & & & & \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Component} & A & x & Ax & I_y & dx & Adx^2 \\
\hline
\text{LEFT} & 3.61 & -1.61 & -5.81 & 8.76 & 1.61 & 9.36 \\
\text{RIGHT} & 3.61 & 1.61 & 5.81 & 8.76 & -1.61 & 9.36 \\
\hline
\text{sums:} & 7.22 & 0.00 & 17.52 & & 18.71 & \\
\hline
X & 0.00 & & & & & \\
I_y & 36.23 & & & & & \\
\hline
\end{array}
\]
7-6: Find $I_x$ and $I_y$ for the cross-sections shown:

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Component} & A & y & Ay & I_x & dy & Ady^2 \\
\hline
\text{LEFT} & 5.87 & 5.00 & 29.35 & 78.90 & 78.90 & 0.00 & 0.00 \\
\text{RIGHT} & 5.87 & 5.00 & 29.35 & 78.90 & 78.90 & 0.00 & 0.00 \\
\hline
\text{sums} & 11.74 & 5.00 & 58.70 & 157.80 & 0.00 & 0.00 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Component} & A & x & Ax & I_y & dx & Adx^2 \\
\hline
\text{LEFT} & 5.87 & -0.61 & -3.56 & 2.80 & 2.80 & 2.18 \\
\text{RIGHT} & 5.87 & 0.61 & 3.56 & 2.80 & 2.80 & 2.18 \\
\hline
\text{sums} & 11.74 & 0.00 & 5.60 & 4.37 & & \\
\hline
\end{array}
\]
7-7: Find the maximum bending stress in the beam and cross-section shown.

\[ \Sigma M_A = 0 = 2k/f(14')(7') + 15k(10') - By(14') \]
\[ By = 24.71k \]
\[ \Sigma f_y = 0 = Ay - 2k/f(14') - 15 + 24.71 \]
\[ Ay = 18.29k \]
\[ M_{\text{max}} = 18.29k(9.15')/2 = 83.68k-f \]
\[ S = I/c = 533.33/8 = 66.67\text{in}^3 \]
\[ f_b = M/S = 83.68k-f[12\text{in}/f]/66.67\text{in}^3 = 15.06\text{ksi} \]
7-8: Find the maximum bending stress in the beam and cross-section shown.

\[
\sum M_A = 0 = 9K(4') + 8K(10') + 7K(18') - By(14')
\]

\[By = 17.29k\]

\[
\sum fy = 0 = Ay - 9 - 8 - 7 + 17.29
\]

\[Ay = 6.71k\]

\[M_{max} = 6.71k(4') + 7k(4') = 54.96k-f\]

\[
S = l/c = 134.67/4 = 33.67in^3
\]

\[f_b = M/S = 54.96k-f(12in/f)/33.67in^3 = 19.59ksi\]
7-9: Find the maximum bending stress in the beam and cross-section shown.

\[ Ay = By = 900\#(7/2) = 3150\# \]
\[ M_{\text{max}} = (3150 + 2250 + 1350 + 450)(2') = 14,400\#-f \]
\[ I_x = \frac{7.25(15^3)}{12} = 2039.06\text{in}^4 \]
\[ S = \frac{I}{c} = \frac{2039.06}{7.5} = 271.87\text{in}^3 \]
\[ f_b = \frac{M}{S} = \frac{14,400\#-f(12\text{in}/f)}{271.87\text{in}^3} = 635.60\text{psi} \]

7-10: Find the maximum bending stress in the beam and cross-section shown.

\[ \sum M_A = 0 = 6k/f(18')(9') - By(12') \]
\[ By = 81k \]
\[ \sum f_y = 0 = Ay - 6k/f(18') + 81 \]
\[ Ay = 27k \]

from shear diagram,
\[ M_{\text{max}} = 36k(6')/2 = 108k-f \text{ at pt. B} \]

W14X90: \[ S_x = 143\text{in}^3 \]
\[ f_b = \frac{M}{S} = \frac{108k-f(12''/f)}{143\text{in}^3} = 9.06\text{ksi} \]
7-11  Find the maximum shear stress in the beam and cross-section shown.

\[
V = 5(16)/2 + 12/2 = 46K \\
Q = 2(6.78)(6.78)/2 = 45.97\text{in}^3 \\
b = 2" \\
f_V = \frac{VQ}{lb} = \frac{46K(45.97)}{314.22/2} = 3.36\text{ksi}
\]

7-12  Find the maximum shear stress in the beam and cross-section shown.

\[
V_{\text{max}} = 5k/(32')/2 = 80k \\
Q = \text{Ady} \\
\text{(under NA:) } 9.7(14)(9.7-9.7/2) = 564.54\text{in}^3 \\
f_V = \frac{VQ}{lb} = \frac{80k(564.54\text{in}^3)}{8669.70\text{in}^4}(14") = 0.372\text{ksi}
\]
7-13  Find the maximum shear stress in the beam and cross-section shown.

\[ V_{\text{max}} = B_y = \frac{[10(4) + 10(10) + 10(18)]}{13} = 24.62k \]

Highest shear stress at edge of middle flange

\[ Q = A_y = 2(1+.5) = 3\text{in}^3 \]
\[ f_v = \frac{VQ}{I_b} = \frac{24.62k(3\text{in}^3)}{10.75(1)} = 6.87\text{ksi} \]

7-14  Find the maximum shear stress in the beam and cross-section shown.

\[ f_v = \frac{V}{twd} \]

W21X55: \( tw = 0.375, \ d = 20.8 \)

\[ B_y = \frac{[3k/f(10')(5') + 10k(15') + 10k(20')]}{20'} = 25k \]
\[ A_y = 3(10) + 10 + 10 = 25k \]

\[ f_v = \frac{V}{twd} = \frac{25k}{[.375(20.8)]} = 3.21\text{ksi} \]
**8-1:** Use deflection charts to find the maximum deflection for the W10X45 beam shown if $E = 29,000\text{ksi}$ and $I = 248$.

**NOTE:** This can be approached by either measuring the deflection at the center of the span OR by factoring the weight the loads and their maximum deflection location.

At midspan:

$x = 7.5$

$\Delta = 5wL^4(1728)/384EI + Pbx(L^2-b^2-x^2)(1728)/6EIL$

$= 5(0.5)(15^4)(1728)/[384(29000)(248)]$

$+ 10(0.5)(7.5)(15^2-5^2-7.5^2)(1728)/[6(29000)(248)(15)] = 0.223''$

**Weighted Loads:**

$x = [0.5(15')(7.5) + 10(\sqrt{10(20)/3})]/[0.5(15) + 10] = 7.88'$

$\Delta = wx(L^3 - 2Lx^2 + x^3)(1728)/24EI + Pbx(L^2-b^2-x^2)(1728)/6EIL$

$= 0.5(7.88)(15^3 - 2(15)(7.88)^2 + 7.88^3)(1728)/[24(29000)(248)]$

$+ 10(0.5)(7.88)(15^2 - 5^2 - 7.88^2)(1728)/[6(29000)(248)(15)] = 0.224''$

**8-2:** Use deflection charts to find the deflection at the end of the overhang for the 7.25" wide by 15" deep beam with $E = 1,200,000\text{psi}$

$I_x = 7.25(15^3)/12 = 2039.06$

$\Delta = -900(2)(12)(3)(14+2)(1728)/[6(1,200,000)(2039.06)(14)]$

$+ 400(3^2)(14+3)(1728)/[3(1,200,000)(2039.06)]$

$= -0.0057'' = 0.0057'' \text{ up}$
8-3: Use deflection charts to find the deflection at the mid-span between supports for the W14X22 beam with E = 29,000 ksi and I = 199.

\[ \Delta = \frac{k}{f(4^2)(6.5)(4(6.5)(13) - 2(6.5^2) - 4^2)(1728)}/[24(29000)(199)(13)] \]
\[ + \frac{2k}{f(6^2)(6.5)(4(6.5)(13) - 2(6.5^2) - 6^2)(1728)}/[24(29000)(199)(13)] \]
\[ = 0.104" \]

8-4: Use the Double Integration Method to find the deflection at the mid-span between supports for the W8X10 beam with E = 29,000 ksi and I = 30.8. Check your answer using deflection charts.

<table>
<thead>
<tr>
<th>A</th>
<th>1 k/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>4'</td>
<td></td>
</tr>
<tr>
<td>3'</td>
<td></td>
</tr>
<tr>
<td>6'</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>2 k/ft</td>
</tr>
</tbody>
</table>

\[ M_x = 2x - 2<x-3> - 2<x-6> \]
\[ \int M_x = dEI \Delta = x^3 - <x-3>x^3 - <x-6>x^3 - 18x = 4.5^3/3 - 1.5^3/3 - 18(4.5) = -51.75 \]
\[ \Delta = 51.75(1728)/[29000(30.8)] = 0.1" \text{ down} \]

using charts:
\[ \Delta = 2\{2(3)(4.5)(81 - 9 - 4.5^2)(1728)\}/[6(29000)(30.8)(9)] = 0.1" \]
8-5: Use the Double Integration Method to find the deflection at the mid-span between supports for the 5.5" X 11.25" beam with \( E = 1,100,000 \text{psi} \). Check your answer using deflection charts.

\[ I_x = 5.5\text{(11.25)}^2/12 = 652.588 \]

By = 1200(15)/12 = 1500
Ay = 1200-1500 = -300

\[ M_x = -300x + 1500<x-12> \]

\[ \int M_x = dEI \Delta = -150x^2 + 750<x-12>^2 + C_1 \]

\[ EI \Delta = -50x^3 + 250<x-12>^3 + C_1x + C_2 = 0 \text{ at } x = 0 \quad \ldots \quad C_2 = 0 \]

& at \( x = 12' \) \ldots -50(1728) + C_1(12) = 0 \quad \ldots \quad C_1 = 7200

\[ \int M_x = dEI \Delta = -150x^2 + 7200 = 0 \text{ at } \Delta_{\text{max}}. \text{ (if } x > 12) \]

\[ -150x^2 + 750x^2 - 18000x + 108000 + 7200 = 600x^2 - 18000x + 115200 \]

\[ = x^2 - 30x + 192 = 0 \quad \ldots \quad x = 15 \pm 7.745 = 9.255' \pm 9.95i \text{ (imaginary number)} \]

Therefore maximum deflection must be at \( x < 12 \)

\[ \int M_x = dEI \Delta = -150x^2 + 7200 = 0 \]

\[ x = \sqrt{(7200/150)} = 6.928' \]

\[ \Delta = [-50x^3 + 7200x](1728)/EI = [-50(6.928^3) + 7200(6.928)](1728)/[1,100,000(652.588)] = 0.08'' \text{ up} \]

Using charts:

\[ \Delta = PaL^2(1728)/(9\sqrt{3EI}) = 1200(3)(12^2)(1728)/(9\sqrt{3}(1100000)(652.588)) = 0.08'' \]
Use the Moment Area Method to find the deflection at $X = 4'$ for the Titanium beam with $E = 15,000$ ksi and $I = 132.4$ in$^4$.

<table>
<thead>
<tr>
<th>Comp.</th>
<th>$Ai$</th>
<th>$x$</th>
<th>$Mi$</th>
<th>$AiMi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$14.33(4)/2 = 28.67$</td>
<td>$4(2/3) = 2.67$</td>
<td>$2.67(.57) = 1.78$</td>
<td>$50.96$</td>
</tr>
<tr>
<td>2</td>
<td>$14.33(4) = 57.33$</td>
<td>$4 + 4/2 = 6$</td>
<td>$2.67 - .33(2) = 2$</td>
<td>$114.67$</td>
</tr>
<tr>
<td>3</td>
<td>$2.67(4/2) = 5.33$</td>
<td>$4 + 4(2/3) = 6.67$</td>
<td>$2.67 - .33(2.67) = 1.78$</td>
<td>$9.48$</td>
</tr>
<tr>
<td>4</td>
<td>$17.33(4)/2 = 26.67$</td>
<td>$8 + 4/3 = 9.33$</td>
<td>$2.67 - .33(4 + 1.33) = 0.91$</td>
<td>$24.3$</td>
</tr>
</tbody>
</table>

$\sum AiMi = 199.41$

$$\Delta = \sum AiMi/EI = 199.41(1728)/(15,000(132.4)) = 0.173\"$$
8-7: Use the Method of Virtual Work to find the deflection at Joint E for the truss shown. The cross-sectional area of each bar is 4in$^2$. 

\[ \Delta = \Sigma P_1 P_2 L/AE = 0.0017" \]
9-1: Design the lightest W12 for the beam shown if \( E = 29,000\text{ksi}, \ F_b = 30\text{ksi}, \ F_v = 20\text{ksi} \) and \( \Delta_{all} = L/240 \).

\[ \sum M_A = 0 = 3k/f(20')(10') + 12k(6') + 12k(14') - By(20') \]
\[ By = 42k \]

\[ \sum f_y = 0 = Ay - 3(20) - 12 - 12 + 42 \]
\[ Ay = 42k \]

\[ V_{\text{max}} = 42k \]

\[ M_{\text{max}} = wL^2/8 + Pa = 3(20)^2/8 + 12(6) = 222k-f \]
\[ \Delta_{\text{max}} = 5wL^4/384EI + Pa(3L^2-4a^2)/24EI = 5(3)(20^4)(1728)/[384(29000)(Ix)] \]
\[ + (12)(6)(2(20)^2 - 4(6)^2)(1728)/(24(29000)(Ix)) = 489.68/Ix \]
\[ \Delta_{all} = L/240 = 20(12)/240 = 1" \]
\[ t_{w,d} \geq V/F_v = 42k/20ksi = 2.1\text{in}^2 \]
\[ S_x \geq M/F_b = 222k-f(12\text{in}/f)/30ksi = 88.8\text{in}^3 \]
\[ 489.68/I_x \leq 1" \quad \cdots \quad I_x \geq 489.68\text{ in}^4 \]

USE W12X 72: \( I_x = 597, S_x = 97.4, t_{w,d} = 0.43(12.3) = 5.289 \)

9-2: Design a 4" wide \( X \ h" \) deep beam with a rectangular cross-section for the beam shown if \( E = 1,200,000\text{psi}, \ F_b = 1800\text{psi}, \ F_v = 180\text{psi} \) and \( \Delta_{all} = L/240 \).

\[ V_{\text{max}} = wL/2 = 1200#/f(16')/2 = 9600# \]

\[ M_{\text{max}} = wL^2/8 = 1200(16)^2/8 = 38,400\#-f \]

\[ \Delta_{\text{max}} = 5wL^4/384EI = 5(1200)(16^4)(1728)/[384(1,200,000)(Ix)] = 1474.56/Ix \]
\[ \Delta_{all} = L/240 = 16(12)/240 = 0.8" \]
\[ 3V/2A \leq F_v \cdots \quad A \geq 3V/2F_v = 3(9600)/2(180) = 80\text{in}^2 \]
\[ S_x \geq M/F_b = 38,400\#-f(12\text{in}/f)/1800\text{psi} = 256\text{in}^3 \]
\[ 1474.56/I_x \leq 0.8" \quad \cdots \quad I_x \geq 1474.56/0.8 = 1843.2\text{ in}^4 \]
\[ 4h \geq 80\text{in}^2 \quad \cdots \quad h \geq 20" \]
\[ 4h^2/6 \geq 256\text{ in}^3 \quad \cdots \quad h \geq 19.60" \]
\[ 4h^3/12 \geq 1843.2\text{ in}^4 \quad \cdots \quad h \geq 17.68" \]

USE 4X20
9-3: Find the most economical W14 for the beam shown if \( E = 29,000 \text{ksi} \), \( F_b = 21.6 \text{ksi} \), \( F_v = 14.4 \text{ksi} \) and \( \Delta_{all} = L/360 \).

\[
M = -5X + 16.28<X-6> - 8<X-8> - 9<X-19>
\]
\[
\int M \, dx = -2.5X^{2}/3 + 8.14<X-6>^{3}/3 - 4<X-8>^{3}/3 - 1.5<X-19>^{3} + C_1X + C_2
\]

Deflection 0 at \( X = 6' \) and 24'.
\[
0 = -180 + 6C_1 + C_2 \quad \ldots \quad C_2 = 180 - 6C_1
\]
\[
0 = -11520 + 15824.16 - 5461.33 - 187.5 + 24C_1 + C_2 = -1344.67 + 24C_1 + C_2
\]
\[
C_1 = 64.7 \quad \text{and} \quad C_2 = 180 - 6(64.7) = -208.2
\]

Deflection is maximum where \( \int M \, dx = 0 \)

If maximum deflection is where \( 8' < X < 19' \),
\[
\int M \, dx = 0 = -2.5X^{2}/3 + 8.14<X-6>^{3}/3 - 4<X-8>^{3}/3 + 64.7(12.06) - 208.2 = -375.02
\]
\[
\Delta = 375.02(1728)/[29000(I_x)] = 22.346/I_x \quad \text{between supports}
\]
\[
\Delta = 208.2(1728)/[29000(I_x)] = 12.4/I_x \text{ at } X = 0
\]

\[
V = 11.28k \quad \text{and} \quad M = 30k-f \text{ from diagrams}
\]
\[
t_{c}d \geq 11.28k/14.4kSI = 0.78in^{2}
\]
\[
S_x \geq 30k-f(12in/f)/21.6kSI = 16.67in^{3}
\]
\[
\Delta_{all} = 18'(12''/f)/360 = 0.6'' \quad \text{between supports}
\]
\[
I_x \geq 22.346/0.6 = 37.24in^{4}
\]
\[
\Delta_{all} = 6'(12''/f)/360 = 0.2'' \quad \text{at overhang}
\]
\[
I_x \geq 12.4/0.2 = 62in^{4}
\]

USE W14X22: \( I_x = 199in^{4} \), \( S_x = 29in^{3} \), \( t_{c}d = 0.23(13.7) = 3.15in^{2} \)
9-4: Design the most economical (lightest weight) HSS rectangular shape for the beam shown if \( E = 29,000 \text{ksi}, F_b = 21.6 \text{ksi}, F_v = 14.4 \text{ksi} \) and \( \Delta_{all} = L/240 \).

\[
\begin{align*}
V &= 20k \\
M &= 20k(12') = 240k-f \\
\Delta &= PL^3/(3EI) = 20(12)^3(1728)/(3(29000)(Ix)) = 686.43/Ix \\
2td &\geq 20k/14.4ksi = 1.39in \quad .. \quad td \geq 0.69in^2 \\
S_x &\geq 240k-f(12in/f)/21.6 = 133.33in^3 \\
\Delta &= 12'(12in/f)/240 = 0.6'' \\
Ix &\geq 686.43/0.6 = 1144.05 \\
\text{USE HSS20X12X3/8:} \quad I_x = 1200in^4, \quad S_x = 120in^3, \quad td = .375(20) = 7.5in^2
\end{align*}
\]

9-5: Find the maximum load, \( P \), the cross-section shown can carry for the beam and loading shown if \( E = 900,000 \text{psi}, F_b = 1600 \text{psi}, F_v = 190 \text{psi} \) and \( \Delta_{all} = L/240 \).

\[
\begin{align*}
V &= P/2 \\
M &= PL/4 = 4P \\
\Delta &= PL^3/(48EI) = P(16)^3(1728)/(48(900,000)(197.27)) \\
&= P/1204.04 \leq 16(12''/f)/240 = 0.8'' \\
P &\leq 1204.04(0.8) = 963.23# \text{ for deflection} \\
S_x &= 197.27/7.06 = 27.94in^3 \\
4P(12in/f)/27.94in^3 &\leq 1600psi \\
P &\leq 931.33# \text{ for flexure} \\
\text{For shear:} \quad Q &= 7.06(1)(7.06/2) = 24.92in^3 \\
VQ/I_xb &= P/2(24.92)/(197.27)(1) = .063P \leq 190psi \\
P &\leq 190/.063 = 3015.87# \text{ for shear} \\
P_{max} &= 931.33#
\end{align*}
\]
10-1: Determine the critical buckling stress and critical buckling load for a 14ft, W14X109 column with pinned ends.

\[ r_y = 3.73'' \]
\[ k = 1.0 \text{ for pinned ends} \]
\[ kL/r = 1.0(14')(12\text{in/f})/3.73 = 45.05 \]

\[ f_{\text{crit}} = \frac{\pi^2 E}{(kL/r)^2} = \frac{\pi^2(29000)}{(45.05)^2} = 141.03\text{ksi} \]

\[ A = 32.0 \text{ in}^2 \]
\[ P_{\text{crit}} = 141.03\text{ksi}(32.0 \text{ in}^2) = 4512.93\text{K} \]

10-2: Given a 4" X 6" (actual dimensions) 10ft wood column with \( E = 1,600,000\text{psi} \):

a) Determine the critical buckling load

\[ r = \sqrt{I/A} = \frac{d}{2\sqrt{3}} = 1.155 \]
\[ kL/r = 1.0(10')(12\text{in/f})/1.155'' = 103.9 \]
\[ f_{\text{crit}} = \frac{\pi^2 E}{(kL/r)^2} = \frac{\pi^2(1,600,000)}{(103.9)^2} = 1462.81\text{psi} \]
\[ P_{\text{crit}} = 1462.81\text{psi}(24\text{in}^2) = 35,107.5\text{#} \]

b) If \( F_{c'} = 1600\text{psi} \), what is the load that will cause the column to crush?

\( 1600\text{psi}(24\text{in}^2) = 38,400\# \)

c) Will the column buckle or crush first? Buckle

10-3: Determine the Critical Buckling Stress of a W21X55 column, with \( E = 29,000\text{ksi} \) and an unbraced length of 20' in the strong direction and 12' in the weak direction.

\[ r_x = 8.40, \quad r_y = 1.73 \]

\[ kLx/r_x = 1.0(20')(12\text{in/f})/8.40'' = 28.57 \]
\[ kLy/r_y = 1.0(12')(12\text{in/f})/1.73'' = 83.24 \quad \text{USE LARGER} \]

\[ f_{\text{crit}} = \frac{\pi^2 E}{(kL/r)^2} = \frac{\pi^2(29,000)}{(83.24)^2} = 41.31\text{ksi} \]
10-4: A 16ft metal column has a hollow circular cross-section with an outside diameter of 18" and a thickness of 1". Which metal will hold more load?
Metal 1 (E = 10,000ksi and Fy = 35ksi) or Metal 2 (E = 12,000ksi and Fy = 25ksi)

\[ A = \pi(9^2) - \pi(8^2) = 53.41 \text{in}^2 \]
\[ I = \pi(9^4)/4 - \pi(8^4)/4 = 1936 \text{in}^4 \]
\[ r = \sqrt{I/A} = \sqrt{1936/53.41} = 6.02 \text{in} \]

\[ kl/r = 16(12)/6.02 = 31.89 \]

**Metal 1:**

\[ Fe = \pi^2(10,000)/(6.02^2) = 2723.37 \text{ksi} \]
\[ 4.71\sqrt{E/Fy} = 4.71\sqrt{10,000/35} = 79.61 \]
\[ 31.89 < 79.61 \quad \text{...} \quad Fcr = \left(\frac{.658Fy}{Fe}\right)Fy = \left(\frac{.658\times35}{2723.37}\right)35 = 34.81 \text{ksi} \]

**Metal 2:**

\[ Fe = \pi^2(12,000)/(6.02^2) = 3268.04 \text{ksi} \]
\[ 4.71\sqrt{E/Fy} = 4.71\sqrt{12,000/25} = 103.18 \]
\[ 31.89 < 103.18 \quad \text{...} \quad Fcr = \left(\frac{.658Fy}{Fe}\right)Fy = \left(\frac{.658\times25}{3268.04}\right)25 = 24.92 \text{ksi} \]

Metal 2 will hold more load.
11-1. For the perimeter in Figure 11.10, design a pattern of support for the perimeter shapes below. Maximum beam spacing is 8' and maximum spacing between columns is 24'.
11-2. For the perimeter in Figure 11.11, frame the outer shape with a maximum beam length of 30' and maximum beam spacing of 10'. Frame the inner shape with a maximum beam length of 60' and a maximum beam spacing of 10'.

ANSWERS WILL VARY>
11-3. Create your own shape to enclose 14,000 – 16,000sf within the limits of a 120’ by 150’ site. Include in your enclosed area a 2000 – 4000sf atrium and frame around it. Maximum beam length = 40’ and maximum beam spacing = 10’.

ANSWERS WILL VARY.

12-1. If LLo = 80psf and tributary area (AT) is 750ft², what is reduced live load on a corner column with a cantilevered slab?

\[ LL = LLo(0.25 + 15/\sqrt{k_{LL}A_T}) = 80psf(0.25 + 15/\sqrt{2(750)}) = 50.98psf \]

12-2. Calculate the design snow load on a flat roof of an office building in Portsmouth, New Hampshire.

\[ pg = \text{ground snow load from ASCE Figure 7-1} = 50 \]
\[ Ce = \text{Exposure Factor from ASCE Table 7-2} = 0.9 \]
\[ Ct = \text{Thermal factor from ASCE Table 7-3} = 1 \]
\[ Is = \text{Importance factor for snow from ASCE Table 7-4} = 1 \]

\[ S = 0.7C_eC_tI_spg = 0.7(0.9)(1)(50) = 31.5psf. \]

12-3. Find the wind loads for Column Line 2 if the fully enclosed structure in Hartford, Connecticut, shown in Figure 12.7 resists wind with column lines 1, 2, and 3. Use Exposure Category D and Occupancy Category III.
1. Risk Category III, \( I_w = 1.15 \) from Table 12.2
2. Wind speed for Hartford = \( V = 130 \text{mph} \)
3. \( K_a = 0.85 \)
4. Exposure Category B
5. \( K_{z1} = 1 \)
6. \( G = 0.85 \)
7. \( GC_{Pi} = -0.18 \)
8. \( K_z \) values from Table 12.6
   \[ \begin{align*}
   @z = 88' & \quad Kh = 0.96 - 2(0.03)/10 = 0.954 \\
   @z = 74Kz & = 0.93 - 6(0.04)/10 = .0906 \\
   @z = 60Kz & = 0.85 \\
   @z = 46Kz & = 0.81 - 4(0.05)/10 = 0.79 \\
   @z = 32Kz & = 0.76 - 8(0.06)/10 = 0.712 \\
   @z = 18Kz & = 0.62 - 2(0.05)/5 = 0.6 \\
   \end{align*} \]
9. \( q = 0.00256Kz(1)(0.85)(130)^2 = 36.77kz \)
10. \( C_p = 0.8 \) for windward walls
11. \( p = q_{z}(GC_{Pi}) - q_{h}(GC_{Pi}) = q_{z}(0.85)(0.8) - 36.77(0.954)(-0.18) = .68q_{z} + 6.314 \)
12. Tributary Area
   \[ \begin{align*}
   @z = 88' & \quad A = 30'(88-74)/2 = 210 \\
   @z = 74A & = 30'(88-60)/2 = 420 \\
   @z = 60A & = 30'(74-46)/2 = 420 \\
   @z = 46A & = 30'(60-32)/2 = 420 \\
   @z = 32A & = 30'(46-18)/2 = 420 \\
   @z = 18A & = 30'(32-0)/2 = 480 \\
   \end{align*} \]

<table>
<thead>
<tr>
<th>Height (f)</th>
<th>( K_z )</th>
<th>( q_z )</th>
<th>( p ) (psf)</th>
<th>( A ) (ft²)</th>
<th>( P ) (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>0.954</td>
<td>53.15</td>
<td>42.45</td>
<td>210.0</td>
<td>8.92</td>
</tr>
<tr>
<td>74</td>
<td>0.906</td>
<td>50.47</td>
<td>40.64</td>
<td>420.0</td>
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<tr>
<td>60</td>
<td>0.850</td>
<td>47.35</td>
<td>38.51</td>
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<td>46</td>
<td>0.790</td>
<td>44.01</td>
<td>36.24</td>
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<tr>
<td>32</td>
<td>0.712</td>
<td>39.67</td>
<td>33.29</td>
<td>420.0</td>
<td>13.98</td>
</tr>
<tr>
<td>18</td>
<td>0.600</td>
<td>33.43</td>
<td>29.04</td>
<td>480.0</td>
<td>13.94</td>
</tr>
</tbody>
</table>
12-4. Determine the seismic loads for column line 2 of the building in Figure 12.7. The site
has very dense soil. The dead loads are 100psf for floors and 50psf for walls. The structure is a
reinforced concrete moment frame.

1. very dense soil = Site Class C
2. $S_1 = 0.20$
3. $S_1 = 0.06$
4. From Table 12.11, $F_a = 1.2$
5. From Table 12.12, $F_v = 1.7$
6. $S_{ds} = (2/3)F_aS_1 = (2/3)(1.2)(0.2) = 0.16$
   $S_{dv} = (2/3)F_vS_1 = (2/3)(1.7)(0.06) = 0.068$
7. $R = 3$ for concrete moment frame
8. $I_E = 1.25$
9. $C_u = 1.7$
10. $C_t = 0.016$, $x = 0.9$ for concrete moment frame
11. $h_n = 88\text{'}$
    \[ T_a = (C_t)/(h_n) = 0.0166 \times (88^{0.9}) = 0.90 = \text{approximate fundamental period.} \]
12. $T = (C_u)/(T_a) = 1.7(0.9) = 1.53$
13. $C_s = S_{ds}(I)/R = 0.16(1.25)/3 = 0.067$
14. $C_{SMN} = 0.01 < C_s = 0.067 \ldots \text{okay}$
15. $C_{SMAX} = S_{dv}/(T(R/I_E)) = 0.068/(1.53(3)/1.25) = 0.0185$
16. $CS = 0.0185$
17. $K = 2$
18. Weight of floors = 100psf(60)(90)/1000 = 540k
   Weight of Walls = 50psf(2)(60+90)(h)/1000#/k = 15h k
   @z = 88\text{'} Wwall = 15(7) = 105k, Wx = 540 + 105 = 645k
   @z = 74\text{'}, 60\text{'}, 46\text{'}, 32\text{' Wwall = 15(14) = 210k, Wx = 540 + 210 = 750k
   @z = 18\text{' Wwall = 15(16) = 240k, Wx = 540 + 240 = 780k

\[ W = 645 + 4(750) = 780 = 4425k \]
\[ C_{VX} = Wxh^2/[645(88^2)] + 750(74^2+60^2+46^2+32^2)+780(18^2) = Wxh^2/14,409,600 \]
\[ V = C_{sW} = .0185(4425) = 81.86 \]
\[ Fx = C_{VX}(V) \]

<table>
<thead>
<tr>
<th>Height (f)</th>
<th>Wx</th>
<th>$h^2$</th>
<th>$Wxh^2$</th>
<th>$C_{VX}$</th>
<th>V</th>
<th>Fx</th>
</tr>
</thead>
<tbody>
<tr>
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<td>645</td>
<td>7744</td>
<td>4,994,880</td>
<td>0.347</td>
<td>81.86</td>
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<td>81.86</td>
<td>1.44</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>= 14,409,600</td>
<td></td>
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</tbody>
</table>
Frame the bay shown if the maximum deck span is a) 8ft. and b) 10ft.
Frame the bay shown if the maximum deck span is a) 8ft. and b) 10ft.
13-3 Frame the bay shown if the maximum deck span is a) 8ft. and b) 10ft.
13-4. Frame a structural floor plan that lies within a 96’ by 144’ rectangle. The plan must include:
1) at least 11,000sf of enclosed space (including the atrium and stairwells listed below).
2) one atrium space between 800 and 1200sf, located anywhere you choose.
3) two 8’ X 20’ stair wells along the perimeter and spaced at opposite ends of the building.
4) Maximum slab span = 12’ = maximum beam spacing
5) Maximum beam span = 40’ = maximum column spacing

Answers will Vary

14-1. For the braced frame shown below, find the additional axial loads in the beams, columns and diagonals caused by the lateral loads. Use the diagonal truss method.
14-2. For the moment frame shown below, find all additional shear, moment and axial forces in all components caused by the lateral loads. Use the Portal Method.
14-3. Determine the additional axial loads on the columns connected to the shear wall shown below if the density of the wall = 90pcf and the wall thickness is 12”.

\[ W = 90 \text{pcf} \times (1')(60')(15')/1000\# / k = 81k \]

\[ \sum M_{TOE} = 0 = 81k(7.5') + 3k(60') + 6k(40') + 6k(20') - Fy(15') \]

\[ Fy = 76.5k \] additional axial loads in the each of the adjacent columns.

14-4. Determine the required thickness of the unconnected shear wall shown if the wall density is 120pcf.

\[ \sum M_{TOE} = 0 = 3k(60') + 6k(40') + 6k(20') - \left[.12(15')(60')t\right](25+20+7.5) - \left[.12(20')(40')(25+10)\right] - \left[.12(25')(20')\right](12.5) = 540 - 5670t - 3360t - 750t \]

\[ t = .055' = 0.66" \]

NOTE: This is not an indicator of thickness required by code or required to support gravity loads. It is only an indicator of the amount of material required to resist overturning.
15-1. Creatively build a tension structure model with a clear height of 2" over the cover zone (4"X10"). Do not extend beyond the site limits (12"X18"), or exceed 6" in height. Compression members and cables may be glued to a base. Compression members must not span the covered area. Draw the concept idea, and the cable and support pattern used.

15-2. Using 1/16" maximum thickness plates only, create a 12" wide structure that can support itself and a full water bottle over a span of 12inches. No plate shall have a length greater than 3 inches measured from any point to any other point on the plate. No adjoining plates may occupy the same plane. As a challenge, include perforations in the design for daylighting from one direction.

15-3. Draw and build a simple space truss to support a full water bottle over a span of 18". The maximum space truss depth is 2". Maximum strut size is 1/16" X 1/16".

15-4. Draw and build a non-orthogonal space truss with varied thickness, varied clear height from base capable of supporting its own weight of a clear span of 18". Maximum strut size = 1/32" X 1/32".

ANSWERS WILL VARY
16-1. Design a series of No. 2 DFL Floor Joists spaced @ 24”o.c., with a moisture content of 20%, termite treatment and a span of 12’. There is a dead load of 15psf and a live load of 40psf.

1. Identify the species of wood: Douglas Fir Larch, NOT southern pine → step 2.

2. Western Species Dimensional Lumber: refer to Tables A.2.2 for sample species

   \[ F_b = 900 \text{psi}, \quad F_v = 180 \text{psi}, \quad E = 1,600,000 \text{psi}, \quad E_{\text{min}} = 580,000 \text{psi}, \quad G = 0.50 \]

3. Assume trial size = 2X12: \( C_F = 1 \), \( A = 16.88 \text{ in}^2 \), \( S = 31.64 \text{ in}^3 \), \( I = 177.98 \text{ in}^4 \)

4. \( F_b' = F_b(C_m)(C_t)(C_{fu})(C_i)(C_r)(2.16)(\lambda) \)

   \[ C_F = 1.0, \quad C_m = 0.85, \quad C_t = 1.0, \quad C_{fu} = 1.0, \quad C_i = 1.15, \quad \lambda = 0.8 \]

   \[ F_b' = (900)(0.85)(1)(1)(1)(0.8)(1.15)(2.16)(0.8)C_L = 1216.17C_L \]

   \[ C_L: \quad d/b = 11.25/1.5 = 7.5 \quad \text{... calculate } C_L: \]

   (code requires blocking at 8’ max.) USE 2 rows of blocking at 48”

   \[ L_u = 48” \quad \text{... } Lu/d = 48/11.25 = 4.27 < 7 \]

   From Table 16.6, \( L_e = 2.06Lu = 2.06(48) = 98.88 \)

   Check that \( R_b^2 = 494.4 \leq 2500. \) Yes ... okay

   \[ E_{\text{min}}' = 591.600 \text{psi} \]

   \[ F_{be} = 1.2(E_{\text{min}}')/R_b^2 = 1.2(591600)/494.4 = 1435.92 \text{psi} \]

   \[ F_b' = 1216.17 \text{psi} \]

   \[ F = F_{be}/F_b' = 1435.92/1216.17 = 1.181 \]

   \[ C_L = (1 + F)/1.9 - \sqrt{((1 + F)/1.9)^2 - (F/0.95)} = 0.875 \]

   \[ F_b' = F_b' C_L = 1216.17(0.875) = 1064.15 \text{psi} \]

5. \( W_{BM} = (.50)(62.4 \text{pcf})[(16.88 \text{in}^2)/(144 \text{in}^2/\text{ft}^2)] = 3.66 \#/\text{f} \)

6. \( W_u = 1.2[15 \text{psf}(24’/12’/\text{ft}) + 3.66 \#/\text{ft}] + 1.6[40 \text{psf}(24’/12’/\text{ft})] = 168.39 \#/\text{f} \)

7. \( M_u = wL^2/8 = 168.39 \# / \text{f} (12’)^2/8 = 3031.02 \#/\text{f} = 36,722.24 \#/\text{-in} \)

8. \( f_b = M/S = 36,722.24 \#/\text{in}/31.64 \text{in}^3 = 1149.57 \text{psi} \)

9. Is \( f_b \leq F_b' \) No. \( 1030.09 < 1313.71 \) ... okay

Try either a 2X14 or a 3X10

3A. Try 3X10: \( C_F = 1.1, \quad A = 23.13 \text{ in}^2, \quad S = 35.65 \text{ in}^3, \quad I = 164.89 \text{ in}^4 \)

4A. \[ C_F = 1.1 \]

   \[ F_b' = (F_b' C_L) = 900(0.85)(1)(1.1)(1.15)(2.16)(0.8)C_L = 1337.79C_L \]

   \[ C_L: \quad d/b = 9.25/2.5 = 3.7 \quad \text{... calculate } C_L: \]

   \[ R_b^2 = Le(d)/b^2 = 98.88(9.25)/2.5^2 = 146.34 \]

   Check that \( R_b^2 = 406.51 \leq 2500. \) Yes ... okay

   \[ F_{be} = 1.2(E_{\text{min}}')/R_b^2 = 1.2(591600)/146.34 = 4851.17 \text{psi} \]

   \[ F_b' = 1337.79 \text{psi} \]

   \[ F = F_{be}/F_b' = 4851.17/1337.79 = 3.626 \]

   \[ C_L = (1 + F)/1.9 - \sqrt{((1 + F)/1.9)^2 - (F/0.95)} = 0.982 \]

   \[ F_b' = F_b' C_L = 1337.79(0.982) = 1313.71 \text{psi} \]

5A. \( W_{BM} = (.50)(62.4 \text{pcf})[(23.13 \text{in}^2)/(144 \text{in}^2/\text{ft}^2)] = 5.01 \#/\text{f} \)

6A. \( W_u = 1.2[15 \text{psf}(24’/12’/\text{ft}) + 5.01 \#/\text{ft}] + 1.6[40 \text{psf}(24’/12’/\text{ft})] = 170.01 \#/\text{f} \)

7A. \( M_u = wL^2/8 = 170.01 \# / \text{f} (12’)^2/8 = 3060.22 \#/\text{f} = 36,722.59 \#/\text{-in} \)

8A. \( f_b = M/S = 36,722.59 \#/\text{in}/35.65 \text{in}^3 = 1030.09 \text{psi} \)

9A. Is \( f_b \leq F_b' \) No. \( 1030.09 < 1313.71 \) ... okay

58
10. Is \( \frac{f_b}{F_b} < 0.9\)? 1030.09%1313.71 = 0.78 BUT smaller size did not work. OKAY for FLEXURE

11. \( C_m = 0.97, C_t = 1, C_i = 0.8 \)
   \[ F_v' = F_v(C_m)(C_t)(C_i)(2.16) = 180(0.97)(1)(0.8) = 241.37\text{psi} \]

12. \( V = \frac{wL}{2} = 170.01\#/(12')/2 = 1020.06\# \)

13. \( f_v = 3V/2A = 3(1020.06\#)/(2[23.13\text{in}^2]) = 66.15\text{psi} \)

14. Is \( f_v \leq F_v' \)? 66.15 < 241.37 . . . okay for shear.

15. \( \Delta_{all} = L(12''/f)/240 = 12'(12''/f)/240 = 0.6'' \)

16. unfactored loads: \( w = 5.01\#/(15\text{psf} + 40\text{psf})(24''/(12''/f)) = 115.01\#/(12') \)

17. \( C_m = 0.9, C_t = 1, C_i = 0.95 \)
   \[ E' = E(C_m)(C_t)(C_i)(E_{min})(1.5) = 1,600,000\text{psi}(0.9)(1)(1.5) = 1,368,000\text{psi} \]

18. \( \Delta_{act} = 5\text{w}L^4/384EI = 5(115.01\#/(12')(1728\text{in}^3/\text{ft}^3))/(384(1,368,000\text{psi}(164.89\text{in}^4))) \)
   = 0.24'' \)

19. Is \( \Delta_{act} \leq \Delta_{all} \)? Yes. 0.24'' < 0.6''

ANSWER: USE 3X10

16-2. Design a series of No. 1 Southern Pine Floor Joists spaced @ 16''o.c., with a moisture content of 18%, and a span of 15'. There is a dead load of 15psf and a live load of 80psf.


20. Southern Pine Dimensional Lumber: Assume trial size = 2X12: \( C_F = 1, \)
    \( A = 16.88\text{in}^2, S = 31.64\text{in}^3, l = 178\text{in}^4 \)

21. \( F_b = 1250, F_v = 175, E = 1,700,000, E_{min} = 620,000 \) and \( G = 0.55. \)

22. \( C_F = 1.0, C_m = 1.0, C_t = 1.0, C_{fu} = 1.0, C_i = 1.0, C_r = 1.15 \)
   \[ F_b' = F_b(C_m)(C_t)(C_i)(2.16)(1.15) = 1,250(1)(1)(1)(2.16)(1.15) = 2484(\text{psi}) \]
   \( d/b = 11.25/1.5 = 7.5 \)

USE 3 ROWS of blocking at 1/4 span, \( L_u = 45'' \), \( L_u/d = 45/11.25 = 4 < 7 \)

23. \( W_{BM} = (0.55)(62.4\text{pcf})(16.88\text{in}^2)/(144\text{in}^2/\text{ft}^2) \) = 4.02#/f

24. \( W_u = 1.2(15(16/12) + 4.02) + 1.6(80(16/12)) = 177.09#/(12') \)

25. \( W_u = 177.09(15^2)(12\text{in}/f)/8 = 59,767.88\#-\text{in} \)

26. \( f_b = 59,767.88/31.64 = 1889\text{psi} \)

27. Is \( f_b \leq F_b' \) yes.
28. Is \( fb/Fb' \geq 0.90? \) \( 1889/1997.14 = 0.945 \) ... yes \text{ OKAY FOR FLEXURE}

29. \( C_m = 1.0, C_t = 1.0, C_i = 1.0 \) ... \( Fv' = 175(2.16)(.8) = 302.4 \text{psi} \)

30. \( V = 177.09(15')/2 = 1328.18 \# \)
31. \( fv = 3V/2A = 3(1328.18)/2(16.88) = 118.03 \text{psi} \)
32. Is \( fv \leq Fv' \)? Yes... \text{ OKAY FOR SHEAR}

33. \( \Delta \text{all} = L(12''/f)/240 = 0.75'' \)
34. Unfactored loads: \( w = (15(16/12) + 4.02) + (80(16/12)) = 130.69 \# /f \)
35. \( C_m = 1.0, C_t = 1.0, C_i = 1.0 \) ... \( E' = E = 1,700,000 \text{psi} \)

36. \( \Delta \text{act} = 5(130.69)(15')(1728)/[384(1,700,000)(178)] = 0.492'' \)
37. Is \( \Delta \text{act} \leq \Delta \text{all} \)? \text{ Yes} ... \text{ USE 2X12}

16-3. Determine how many Select Structural Southern Pine 2X12’s must be joined together to support a factored load of 300\#/f over a span of 18’.

Find required \( Sx, Ix, A \) and then divide by values for one 2X12 to find number required.
1. Southern Pine \( \rightarrow \) step 20.

20. Southern Pine Dimensional Lumber: Assume trial size = 2X12: \( C_F = 1, \)
\( A = 16.88 \text{ in}^2, S = 31.64 \text{in}^3, l = 178 \text{in}^4 \)
21. \( F_b = 1900, F_v = 175, E = 1,800,000, E_{\text{min}} = 660,000 \) and \( G = 0.55. \)
22. \( C_F = 1.0, C_m = 1.0, C_t = 1.0, C_{fu} = 1.0, C_i = 1.0, C_r = 1 \)
\( F_{b'} = F_b(C_m)(C_t)(C_i)(2.16)(\lambda) = 1900(1)(1)(1)(1)(1)(1.5) = 3283.2 \text{psi} \)
\( \lambda = 11.25/1.5 = 7.5 \)
\( \text{USE 5 ROWS of blocking at 1/6 span}, \text{ } Lu = 36'', \text{ } Lu/d = 36/11.25 = 3.2 < 7 \)
\( E = 2.06Lu = 2.06(36) = 74.16 \)
\( R_b = 74.16(11.25)/(1.5)^2 = 370.8 \)
\( 370.8 \leq 2500 \text{ ... okay} \)
\( C_m = 1.0, C_i = 1.0, E_{\text{min}} = 660,000 \text{psi} \)
\( E_{\text{min}}' = E_{\text{min}}(C_m)(C_t)(1.5) = 660,000(1)(1)(1.5) = 990,000 \text{psi} \)
\( F_{bE} = 1.2(E_{\text{min}}')/R_b = 1.2(990,000)/370.8 = 3203.88 \text{psi} \)
\( F_b = F_{bE}C_L \) \( F_{b*} = 3283.2 \)
\( F = F_{bE}/F_b = 3203.88/3283.2 = 0.976 \)
\( C_L = (1 + F)/1.9 - \sqrt{[(1 + F)/1.9]^2 - (F/0.95)]} = 0.807 \)
\( F_{b*} = .807(3283.2) = 2649.54 \)
23. \( W_{BM} = (.55)(62.4 \text{pcf})(16.88 \text{in}^2)/(144 \text{in}^2/\text{ft}^2) = 4.02\# /f \)
24. \( W_u = 1.2(40.02) + 300\# /f = 304.82\# /f \)
25. \( M_u = 304.82(18^2)(12\text{in}/f)/8 = 148,142.52\# - \text{in} \)
26. \( f_b = 148,142.52/31.64 = 4682.13 \text{psi} \)
27. Is \( fb \leq Fb' \)? No! Estimate new \( Sx = 31.64(4682.13/2649.54) = 55.9 \text{in}^3 \)
\( \text{TRY 2-2X12 } Sx = 31.64(2) = 63.28 \)

22A. \( C_F = 1.0, C_m = 1.0, C_t = 1.0, C_{fu} = 1.0, C_i = 1.0, C_r = 1 \)
The document contains calculations related to structural engineering, specifically involving the design of a beam. The calculations include the determination of bending stress, shear, and deflection, as well as the calculation of factored compressive load. The text is dense with mathematical expressions and units, typical of engineering reports. The calculations are methodical, involving various factors such as the modulus of elasticity, deflection, and various coefficients. The document concludes with a finding that an 8' No. 1 Southern Pine 2x6 can support without bracing.
7. $F_{ce} = 0.822\frac{(E_{min})}{(Le/d)^2} = 0.822(930,000)/64^2 = 186.64\text{ psi}$
8. $F = F_{ce}F_{c} = 186.64/3628.8 = 0.051$
9. $c = 0.8$ for sawn lumber,
10. $C_{p} = \frac{(1 + F)}{2c} - \left(\frac{(1 + F)}{2c}\right)^2 - \frac{(F/c)}{1.6} = 1.051/1.6 - ((1.051/1.6)^2 - 0.051/0.8)^1/2 = 0.0504$
11. $F_{c}' = F_{c}C_{p} = 3628.8(0.0504) = 182.89\text{ psi}$
12. $P = F_{c}'A = 182.89(1.5')(5.5'') = 1508.84\#$

16-5. Determine the maximum unbraced length of the box column shown in Figure 16.7, subjected to a load of 2000\# if the 2X6’s are No. 2 DFL and there is incising and a moisture content of 21%.

\[F_{c}' = \frac{fc = P/A}{2000/[4(1.5)(5.5)]} = 60.606\text{ psi}\]
\[F_{c}' = 60.606\text{ psi} = F_{c}C_{p}\]

No.2 DFL: $F_{c} = 1350\text{ psi}$, $E_{min} = 580,000\text{ psi}$
$C_{F} = 1.2$ for a 2X6
$C_{m} = 0.8$, $C_{t} = 1$, $C_{i} = 0.8$
$F_{c} = 1350(1.2)(0.8)(0.8)(2.16)(0.8) = 1791.59\text{ psi}$
$F_{c}' = 60.606\text{ psi} = 1791.59(C_{p})$
$C_{p} = 60.606/1791.59 = 0.0338 = (1+F)/1.6 - \sqrt{((1+F)/1.6)^2 - F/c}
F = 0.0341 = F_{ce}/F^{*}$
$F_{ce} = 0.0341(1791.59) = 61.093 = 0.822E_{min}/(Le/d)^2$
$E_{min}' = 580,000(0.9)(0.95)(1.5) = 743,850\text{ psi}$
$61.407 = 0.822(743,850)/(Le/5.5)^2$
$Le = 5.5'[822(743,850)/61.093]^{1/2} = 558.24''$
however, $(Le/d) \leq 50 ... 50(5.5'') = 275'' \text{ limit}$

16-6. A 2X12 joist carrying a factored load of 600#/f and having a span of 14’ fully bears on a flat 2X6 No. 2 SP top plate. Is this acceptable?

$Lb = 1.5''$
$C_{b} = (Lb + 0.375)/Lb = (1.5+0.375)/1.5 = 1.25$

$F_{c} = F_{c}(C_{m})(C_{t})(C_{i})(C_{b})(1.5) = 565(1.25)(1.5) = 1059.38\text{ psi}$
$A = 1.5(5.5') = 8.25\text{ in}^2$
$P/A = 600/8.25 = 72.73\text{ psi} < 1059.39\text{ psi} ... \text{ okay for 2X6}$

$Lb_2 = 5.5''$, $C_{b} = (5.5+0.375)/5.5 = 1.068$
$F_{c} = 565(1.068)(1.5) = 905.13 > 72.73 ... \text{ okay for 2X12.}$
16-7. What is the partial bearing length required for a No. 2 So. Pine 2X10 bearing on a flat No. 2 So. Pine 2X4 with a Factored Load of 2000#?

\[
P/A = 2000/(1.5L_b) = Fc' = 565(C_b)(1.5) = 565((L_b+.375)/L_b)(1.5)
\]

\[
2000/1.5 = 565(1.5)(L_b+.375) \ldots L_b = 1.198''
\]

16-8. Find the maximum allowable Tension in a No. 2 So Pine 2X4 with 18% moisture content & at room temperature.

\[
F_t = 825 \text{psi} \quad C_F = 1.5
\]

\[
F_t' = F_t(C_t)(C_m)(C_i)(C_f)(2.16)(\lambda) = 825(1.5)(2.16)(.8) = 2138.4 \text{psi}
\]

\[
T = A F_t' = 1.5(3.5)(2138.4) = 11,226.6##
\]

16-9. A 3X5.5 column, built up using 2-2X6s of No. 2 Southern Pine, is 16' long with one end fixed and the other pinned. It has a factored axial load of 3000#, a factored Mx of 750 #-in and My of 150 #.in. Is this column adequate?

1. \(F_b = 1250\text{psi, } F_c = 1600\text{psi, } \text{Emin} = 580,000\text{psi}\)
2. Find section properties: \(A = 2(8.25) = 16.5\text{in}^2, S_x = 2(7.56) = 15.12\text{in}^3, \)
\(I_y = \sum y_i + \sum A y_i^2 = 2(1.55) + 2(8.25)(.75)^2 = 12.38\text{in}^4, c = 1.5'' \ldots S_y = 12.38/1.5 = 8.25\text{in}^3\)
3. \(f_c = P/A = 3000/16.5 = 181.82\text{psi}\)
\(f_{b1} = M_x/S = 750/15.12 = 49.60\text{psi}\)
\(f_{b2} = M_y/S = 150/8.25 = 18.18\text{psi}\)
4. Find \(F_{CE1}, F_{CE2}: \)
\(\text{Emin}' = 580,000(1.5) = 870,000\text{psi}\)
\(L_u = 16ft(12'') = 192'' \quad L_e = kL = 0.8(192) = 153.6''\)
\(L_e/d_1 = 153.6/5.5 = 27.93\)
\(F_{CE1} = 0.822(870,000)/27.93^2 = 916.75\text{psi}\)
\(F_{CE1} = 916.75 > 181.82 = f_c \quad \text{okay}\)
\(L_e/d_2 = 153.6/3 = 51.2\)
\(F_{CE2} = 0.822(870,000)/51.2^2 = 272.80\)
\(F_{CE2} = 272.80 > 181.82 = f_c \quad \text{okay}\)
5. Find \(C_p: Fc' = F_c(C_i)(C_p)(2.16)(\lambda) = 1600(1.0)(2.16)(.8)C_p = 2764.8C_p\)
\(F_{CE1}/F_c' = 916.75/2764.8 = 0.332 = F\)
\(C_{p1} = (1 + F)/2c - [(1 + F)/2c]^2 - (F/c)]^{1/2} = (1.332/1.6) - [(1.332/1.6)^2 - (0.332/0.8)]^{1/2} = 0.305\)
\(F_{CE2}/F_c' = 272.80/2764.8 = 0.099\)
\(C_{p2} = (1 + F)/2c - [(1 + F)/2c]^2 - (F/c)]^{1/2} = (1.099/1.6) - [(1.099/1.6)^2 - (0.099/0.8)]^{1/2} = 0.097\)

Use lesser value of \(C_p = 0.097\)
6. Check compression: \(F_c' = 0.097(2764.8) = 268.19\text{psi} > 181.82 = f_c \quad \text{okay for compression}\)
7. Find \(C_p, F_{b1}', F_{b2}':\)
\(F_{b1}' = F_b(C_i)(C_p)(2.16)(\lambda) = 1250(1.0)(2.16)(.8)C_p = 2160C_p\)
\(L_e = 1.84L_u \quad \text{(equal end moments)} = 1.84(192) = 353.28\)
\[
R_b^2 = \frac{L_e d_1}{d_2^2} = 353.28(5.5)/3^2 = 215.89 \\
F_{bE1} = 1.2(870,000)/215.89 = 4835.80 \\
F_{bE1}/F^* = 4835.80/2160 = 2.239 \\
C_{L1} = (1 + F)/1.9 - \sqrt{[(1 + F)/1.9]^2 - (F/0.95)} = (3.239/1.9) - \sqrt{(3.239/1.9)^2 - (2.239/0.95)} = 0.964 \\
F_{b1} = 0.964(2160) = 2081.37 \text{ psi} > 49.60 \text{ okay} \\
R_b^2 = \frac{L_e d_1}{d_2^2} = 353.28(3)/5.5^2 = 35.04 \\
F_{bE2} = 1.2(870,000)/35.04 = 29,794.52 \\
F_{bE2}/F^* = 29,794.52/2160 = 13.79 \\
C_{L2} = (1 + F)/1.9 - \sqrt{[(1 + F)/1.9]^2 - (F/0.95)} = (14.79)/1.9 - \sqrt{(14.79/1.9)^2 - (13.79/0.95)} = 0.996 \\
F_{b2} = 0.996(2160) = 2151.36 > 18.18 \text{psi} = f_{b2} \text{ okay} \\
8. F_{bE} = \text{lesser of } F_{bE1} \text{ and } F_{bE2}; \quad F_{bE} = 2081.37 \text{ psi} \\
9. \frac{[f_c/Fc']^2 + f_{b1}/[F_{b1}^* [1 - (fc/Fc')]] + f_{b2}/[F_{b2}^* [1 - (fc/Fc') - (f_{b1}/F_{bE})^2]]}{[181.82/(268.19)^2 + 49.60/(2081.37)[1 - 181.82/(916.75)] + 18.18/(2151.36)[1 - 181.82/272.8 - (49.60/2081.37)^2]} = 0.339 + 0.030 + 0.025 = 0.394 < 1.0 \text{ okay} \\
16-10. \text{ Check the adequacy of a 2X8 dimensional lumber beam with } L = 12', \text{ } L_u = 4', \text{ with two concentrated loads of } 2000\# \text{ at } 4' \text{ o.c. and a tension load of } 500\#, \text{ using no. 1 DFL.} \\
1. \text{ Check Flexure:} \\
\text{Western Species Dimensional Lumber: refer to Tables A.2.2 for sample species} \\
\text{ } F_b = 1000\text{psi, } F_t = 675\text{psi, } F_v = 180\text{psi, } E = 1,700,000\text{psi, } E_{\min} = 620,000\text{psi, } G = 0.5 \\
\text{2X8: } C_e = 1.2 \text{ for } F_b \text{ and } F_t, \quad A = 10.88 \text{ in}^2, \quad S = 13.14 \text{in}^3, \quad I = 47.63 \text{in}^4 \\
\text{Fb'} = F_b(C_m)(C_t)(C_i)(1.5) = 1000(1)(1)(1.5) = 1500 \text{psi} \\
\text{CL: } d/b = 7.25/1.5 = 4.833 \ldots \text{ calculate CL because there is no mention of full sheathing and blocking at ends.} \\
1. \text{ From Table 16.6, } L_e = 1.68L_u = 1.68(48) = 80.64'' \\
2. \text{Rb}^2 = L_e d_1/b_2^2 = 80.64(7.25)/1.5^2 = 259.84 \\
3. \text{Check that } Rb^2 \leq 2500. \text{ Yes ... okay} \\
4. \text{Emin'} = Emin(C_m)(C_t)(C_i) = 620,000(1)(1.5) = 930,000\text{psi} \\
5. \text{F}_{be} = 1.2(E_{\min'})/Rb^2 = 1.2(930,000)/259.84 = 4294.95\text{psi} \\
6. \text{Fb'} = 2073.6\text{psi} \\
7. \text{F} = F_{be}/Fb' = 4294.95/2073.6 = 2.07 \\
8. \text{CL} = (1 + F)/1.9 - \sqrt{[(1 + F)/1.9]^2 - (F/0.95)} = 0.959 \\
\text{Fb'} = Fb^* (CL) = 2073.6(0.959) = 1988.58\text{psi} \\
\text{Find weight of beam: } W_{BM} = (\text{specific gravity})(62.4\text{pcf})(Ain^2)/(144in^2/ft^2) = .5*62.4*10.88/144 = 2.36\#/	ext{f} \\
\text{Find Factored Loads: Assume points loads are live loads (worse case)} \\
Wu = 1.2(DL) + 1.6(LL) = 1.2[2.36\#/f] = 2.83\#/f; \quad Pu = 2000(1.6) = 3200\# \\
M = wL^2/8 + PL/3 = 2.83(12)^2(12in/ft)/8 + 3200(12)(12)/3 = 153,560.94\text{#-in}
fb = M/S = 153560.94/13.14 = 11,693.37psi > 1988.58psi  NG for flexure

This is the end of the problem as stated.

... Try W4X16

4X16:  C_f = 0.9 for F_{b} and F_{t}, A = 53.38 in^2, S = 135.66 in^3, I = 1034.42 in^4

Fb' = Fb(C_m)(C_f)(C_{L})(C_f)(2.16)/(A) = 1000(1)(1)(C_{L})(0.9)(2.16)(0.8) = 1555.2 C_{L}

CL:  d/b = 15.25/3.5 = 4.36  ... calculate CL  
1. From Table 16.6, Le = 1.68Lu = 1.68(48) = 80.64"
2. Rb^2 = Le(d)/b^2 = 80.64(15.25)/3.5^2 = 100.39
3. Check that Rb^2 = 100.39 ≤ 2500. Yes ... okay
4. Emin' = Emin(Cm)(C_t)(Cl)(1.5) = 620,000(1)(1)(1)(1.5) = 930,000psi
5. F_{bE} = 1.2(Emin')/Rb^2 = 1.2(930,000)/100.39 = 11,116.65psi
6. Fb* = 1555.2psi
7. F = F_{bE}/Fb* = 11,116.65/1555.2 = 7.15
8. CL = 1 + F/1.9) - \sqrt{[1 + F/1.9)^2 - (F/0.95)]} = 0.992

Fb' = Fb* (CL) = 1555.2(0.992) = 1542.77psi

Find weight of beam:  W_{BM} = (specific gravity)(62.4pcf)([Ain^2]/[144in^2/ft^2]) = .5*62.4*53.38/144 = 11.57#/ft

Find Factored Loads: Assume points loads are live loads (worse case)

Wu = 1.2[DL] + 1.6[LL] = 1.2[11.57#/f] = 13.88#/f; Pu = 2000(1.6) = 3200#

M = wL^2/8 + PL/3 = 13.88(12)^2(12in/ft)/8 + 3200(12)(12)/3 = 153,849.84#-in

fb = M/S = 153849.84/135.66 = 1134.08psi < 1542.77psi  OKAY for flexure

2. Check Tension:

Ft' = 675(0.9)(2.16)(0.8) = 1049.76psi

ft = P/A = 500/53.38 = 9.37psi < 1049.76psi okay for tension

3. Check flexure and tension combined:

ft/Ft + fb/Fb* = 9.37/1049.76 + 1134.08/ 1542.77 = 0.74 < 1.0  okay

4. Check Shear:

Fv' = Fv(C_m)(C_t)(2.16)/(2.16) = 180(1)(1)(2.16)(0.8) = 311.04

V = 11.57(12)/2 + 3200 = 3269.42

fv = 3V/2A = 3(3269.42)/(2(53.38)) = 91.87 < 311.04psi ... okay for shear

5. Check Deflection:

\Delta_{all} = L/240 = 12(12)/240 = 0.68"

E' = E(C_m)(C_t) = 1,700,000(1)(1) = 1,700,000 psi

l = 1034.42 in^4

unfactored load:  P = 2000,  W = 11.57#/f

\Delta_{max} = 5wL^4/384EI + 23PL^3/648EI = 5(11.57)(12)^4(1728)/384/1700000/1034.42 + 23(2000)(12)^3(1728)/648/1700000/1034.42 = 0.124 < 0.6 okay.
17-1. Design the most efficient 20’ long timber beam of No. 2 Douglas-fir-Larch 8X2 with a uniform dead load, \( w_d = 30 \#/f \) and a uniform live load \( w_l = 640 \#/f \) with full lateral bracing.

1. \( F_b = 875 \text{psi}, \ F_v = 170 \text{psi}, \ E = 1,300,000 \text{psi}, \ E_{min} = 470,000 \text{psi}, \ G = 0.5 \)
2. Assume factored beam weight = \( L(10) = 20(10) = 200 \#/f; \ d < 12'' \)
3. \( F_b' = F_b(C_l)(C_o)(C_t) = 875(1)(1)(0.8) = 875(1)(1)(0.8) = 1512 C_l \)
   Assume \( C_l = 1 \), \( F_b^* = 1512 \text{psi} \)
4. \( W_u = 200 + 1.2(30) + 1.6(640) = 1260 \#/ft \)
5. \( M = wL^2/8 = 1260(20)^2(12 \text{in/ft})/8 = 756,000 \#-\text{in} \)
6. \( S_{req} \geq M/F_b' = 756,000/1512 = 500 \text{in}^3 \)
7. Choose size based on \( S_{req} \): Try 8X22 \( A = 152.25 \text{in}^2, \ S = 532.88 \text{in}^3, I_x = 5595.19 \text{in}^4 \)
8. \( C_l = 1 \) (full lateral bracing) \( F_b^* = F_b^* (C_l) = 1512(1) = 1512 \text{psi} \)
9. \( C_f = 0.94 \)
10. \( F_b^* = 1512(0.94) = 1421.28 \text{psi} \)
11. Find actual weight of beam: \( w_{bm} = (\text{specific gravity})(62.4 \text{pcf})(A/144) = 0.5(62.4)(152.25)/144 = 32.99 \#/f \)
12. \( W_u = 1.2(30) + 1.6(640) + 1.2(32.99) = 1099.59 \#/f \)
13. \( M_u = wL^2/8 = 1099.59(20)^2(12 \text{in/ft})/8 = 659,754 \#-\text{in} \)
14. \( f_v = M_u/S = 659,754/532.88 = 1238.09 \text{psi} \)
15. Is \( f_v \leq F_b^* \? Yes \rightarrow \text{step 16} \)
16. Is \( f_v/F_b^* \geq 0.90 \? No, 1238.09 < 1421.28 \text{psi} \)
17. \( F_v' = F_v(C_l)(0.8) = 170(1)(0.8) = 293.76 \text{psi} \)
18. \( V = wL/2 = 1099.59 \#/f(20'/2) = 1099.59 \# \)
19. \( f_v = 3V/2A = 3(1099.59\#/f)[2(152.25 \text{in}^2)] = 108.33 \text{psi} \)
20. Is \( f_v \leq F_v' \? Yes \ 108.33 \text{psi} < 293.76 \text{psi} \ ... \ okay \ for shear. \)
21. \( \Delta_all = L(12''/f)/240 = 20(12)/240 = 1.0'' \)
22. \( w = 32.99 + 30 + 640 = 702.99 \#/ft \)
23. \( E' = E_{min}/1.3 = 1,300,000 \text{psi} \)
24. \( \Delta_{act} = 5wL/384E' = [5(702.99)(20^2)(1728)]/384/1,300,000/5595.19 = 0.348'' \)
25. Is \( \Delta_{act} \leq \Delta_{all} \? Yes \ 0.348'' < 1.0'' \ ... \)

ANSWER: USE 8X22
17-2. Design the most efficient, 16' long timber beam of No. 1 Douglas-fir-Larch 6X_ a uniform dead load, \( w_D = 20\# / f \) and a uniform live load \( w_L = 600\# / f \) with lateral bracing at 4' o.c..

1. \( F_b = 1000\psi \), \( F_v = 180\psi \), \( E = 1,700,000\psi \), \( Emin = 620,000\psi \), \( G = 0.5 \)
2. Assume factored beam weight = \( L(10) = 16(10) = 160\# / f \); \( d < 12'' \)
3. \( F_{b*} = F_b(C_l)(C_t)(C_i)(C_{fu})(C_i)(2.16)(0.8) = 1000(1)(C_l)(1)(C_{fu})(0.8) = 1728CL \)
   Assume \( C_l = 1 \), \( F_{b*} = 1728\psi \)
4. \( Wu = 160 + 1.2(20) + 1.6(600) = 1144\# / f \)
5. \( M = w_L^2 / 8 = 1144(16)^2(12in / f) / 8 = 439,296\# - in \)
6. \( S_{req} \geq M / F_{b*} = 439,296\# - in / 1728 = 254.22in^3 \)
7. Try 6X18 \( A = 93.5in^2 \), \( S = 264.92in^3 \), \( I_x = 2251.79in^4 \).
8. \( d/b = 17 / 5.5 = 3.09 \) ... find \( C_L \) \( Lu = 48'' \), \( Lu/d = 48 / 17 = 2.82 \)
   1. From Table 16.6, \( Le = 2.06Lu = 2.06(48) = 98.88'' \)
   2. \( R_b^2 = Le(d) / b^2 = 98.88(17) / 5.5^2 = 55.57 \)
   3. Check that \( R_b^2 = 55.57 \leq 2500. \) Yes ... okay
4. \( Emin^* = Emin(C_m)(C_t)(C_l)(1.5) = 620,000(1)(1)(1)(1.5) = 930,000\psi \)
5. \( F_{be} = 1.2(Emin^*) / R_b^2 = 1.2(930,000) / 55.57 = 20,827.88\psi \)
6. \( F_{b*} = 1728\psi \)
7. \( F = F_{be} / F_{b*} = 20,827.88 / 1728 = 11.62 \)
8. \( CL = (1 + F) / 1.9 - \sqrt{((1 + F) / 1.9)^2 - (F/0.95)} = 0.998 \)
   \( F_{b*} = F_{b*} (CL) = 1728(0.998) = 1724.54\psi \)
9. \( C_{s} = 0.96 \)
10. Adjust \( F_{b*} \) for new CF: \( F_{b'} = 1724.54(0.96) = 1655.56\psi \)
11. \( W_{bm} = \) [specific gravity](62.4pcf)[A / 144] = .5(62.4)(93.5) / 144 = 20.26\#/f
12. \( Wu = 1.2(20.26) + 1.2(20) + 1.6(600) = 1008.31 \)
13. \( Mu = wL^2 / 8 = 1008.31(16)^2(12in / f) / 8 = 387,191.04\# - in \)
14. \( fb = Mu / S = 387,191.04 / 264.92 = 1461.54\psi \)
15. is \( fb \leq F_{b'} \)? Yes \( \rightarrow \) step 16 \( 1461.54 < 1655.56\psi \)
16. Is \( fb / F_{b'} \geq 0.909 \) \( \) No, \( 1461.54 / 1655.56 = 0.88 \) Try next smaller size.
7A. Try 6X16 \( A = 82.5in^2 \), \( S = 206.25in^3 \), \( I_x = 1546.88in^4 \).
8A. \( d/b = 15 / 5.5 = 2.73 \) ... find \( C_L \) \( Lu = 48'' \), \( Lu/d = 48 / 17 = 2.82 \)
   1. From Table 16.6, \( Le = 2.06Lu = 2.06(48) = 98.88'' \)
   2. \( R_b^2 = Le(d) / b^2 = 98.88(15) / 5.5^2 = 49.03 \)
   3. Check that \( R_b^2 = 49.03 \leq 2500. \) Yes ... okay
4. \( Emin^* = Emin(C_m)(C_t)(C_l)(1.5) = 620,000(1)(1)(1)(1.5) = 930,000\psi \)
5. \( F_{be} = 1.2(Emin^*) / R_b^2 = 1.2(930,000) / 49.03 = 22,761.57\psi \)
6. \( F_{b*} = 1728\psi \)
7. \( F = F_{be} / F_{b*} = 22,761.57 / 1728 = 13.17 \)
8. \( CL = (1 + F) / 1.9 - \sqrt{((1 + F) / 1.9)^2 - (F/0.95)} = 0.996 \)
   \( F_{b*} = F_{b*} (CL) = 1728(0.996) = 1721.09\psi \)
9A. \( C_{s} = 0.98 \)
10A. Adjust \( F_{b'} \) for new CF: \( F_{b'} = 1721.09(0.98) = 1686.67\psi \)
11A. Find actual weight of beam: \( W_{bm} = .5(62.4)(82.5) / 144 = 17.88\#/f \)
12A. \( Wu = 1.2(17.88) + 1.2(20) + 1.6(600) = 1005.46\#/f \)
13A. \( Mu = wL^2 / 8 = 1005.46(16)^2(12in / f) / 8 = 386,096.64\# - in \)
14A. \( fb = Mu / S = 386,096.64 / 206.25 = 1871.98\psi \)
15A. is \( fb \leq Fb' \)? NO, \( 1871.98 > 1721.09 \) ... No GOOD, go back to 6X18
17. \( Fv' = 180(1)(2.16)(.8) = 311.04 \text{psi} \)
18. \( V = \frac{wL}{2} = \frac{1005.46}{16/2} = 8043.68 \# \)
19. \( fv = \frac{3V}{2A} = \frac{3(8043.68\#)}{[2(93.5in^2)]} = 129.04 \text{psi} \)
20. is \( fv \leq Fv' \)? yes \( 129.04 \text{psi} < 311.04 \text{psi} \) ... okay for shear.
21. \( \Delta \text{all} = \frac{L(12''/f)}{240} = \frac{136(12)}{240} = 0.8'' \)
22. \( w = 17.88 + 20 + 600 = 637.88\#/f \)
23. \( E' = 1,700,000 \text{psi}(1) = 1,700,000 \text{psi} \)
24. \( \Delta \text{act} = \frac{5wL}{384EI} = \frac{5(637.88)(16^4)(1728)}{384/1,700,000/2251.79} = 0.245'' \)
25. is \( \Delta \text{act} \leq \Delta \text{all} \)? Yes→ \( 0.245'' < 0.8'' \) ... OKAY for deflection
ANSWER: USE 6X18

17-3. Design a square Select Structural DFL Column 16' long to carry a factored axial load of 80,000# with fixed connections and a moisture content of 16%.

1. \( Fc = 1150 \text{psi} \) and \( Emin = 580,000 \text{psi} \).
2. \( Fc' = Fc(C_m)(C_i)(C_p)(2.16)(\lambda) = Fc* \) Assume \( C_p = 1 \) and \( C_i = 1 \) for now. \( Fc' = 1150(1)(1)(2.16)(.8) = 1987.2(\text{psi}) \) ... \( Fc* = 1987.2 \text{psi} \)
3. \( L_e = kL(12''/f) = 0.65(16')(12''/f) = 124.8'' \)
\( d_{min} = b_{min} = 124.8''/50 = 2.50'' \)
4. Atrial = \( P/Fc* = 80,000#/1987.2 = 40.26\text{in}^2 \)
Try 8X8: \( A = 52.56\text{in}^2, d = b = 7.25'' \)
5. Use larger of \( L_e/d = 124.8''/7.25 = 17.21 \)
6. \( Emin' = 580,000(1)(1.5) = 870,000 \text{psi} \)
7. \( F_{Ce} = 0.822(Emin')/(Le/d)^2 = 0.822(870,000)/17.21^2 = 2414.51 \text{psi} \)
8. \( F = F_{Ce}/Fc* = 2414.51/1987.2 = 1.215 \)
9. \( c = 0.8 \) for sawn lumber,
10. \( C_p = (1+F)/2c - [(1+F)/2c]^2 - (F/c)]^{1/2} = 0.754 \)
11. \( Fc' = Fc*(C_p) = \text{allowable compressive stress} = 1987.2(0.754)(1) = 1498.35 \text{psi} \)
12. \( fc = P/A = \text{actual compressive stress} = 80,000#/52.56\text{in}^2 = 1522.07 \text{psi} \)
13. Is \( fc < Fc' \)? NO. \( 1522.07 \text{psi} > 1498.35 \text{psi} \) Try next large size: 10X10

Try 10X10: \( A = 85.56\text{in}^2, d = b = 9.25'' \)
5. Use larger of \( L_e/d = 124.8''/9.25 = 13.49 \)
6. \( Emin' = 580,000(1)(1.5) = 870,000 \text{psi} \)
7. \( F_{Ce} = 0.822(Emin')/(Le/d)^2 = 0.822(870,000)/13.49^2 = 3929.77 \text{psi} \)
8. \( F = F_{Ce}/Fc* = 3929.77/1987.2 = 1.978 \)
9. \( c = 0.8 \) for sawn lumber,
10. \( C_p = (1+F)/2c - [(1+F)/2c]^2 - (F/c)]^{1/2} = 0.865 \)
11. \( Fc' = Fc*(C_p) = \text{allowable compressive stress} = 1987.2(0.865)(1) = 1718.93 \text{psi} \)
12. \( fc = P/A = \text{actual compressive stress} = 80,000#/85.56\text{in}^2 = 935.02 \text{psi} \)
13. Is \( fc < Fc' \)? YES. \( 935.02 \text{psi} < 1718.93 \text{psi} \)
Try next large size: 10X10
ANSWER: USE 10X10
17-4. Design a 6X_{No.2} Southern Pine Column 12’ long to carry a factored axial load of 90,000# with pinned connections and bracing at 4’ from top in the weak direction.

1. $F_c = 525 \text{psi}$ and $E_{min} = 440,000 \text{psi}$.
2. $F_{c'} = F_c(C_m)(C_l)(C_p)2.16 \ (\lambda) = 525(2.16)(.8)(C_p) = 907.2C_p$
   
   Assume $C_m = 1$ and $C_l = 1$ for now.
3. $L_{ex} = kL(12’/f) = 1(12')(12'/f) = 144'' \ L_{ey} = 1(8')(12'/f) = 96''$
   
   $d_{min} = 144''/50 = 2.88, \ b_{min} = 96''/50 = 1.92''$
4. Atrial = $P/F_c^* = 90,000#/907.2 = 99.21 \text{in}^2$
   
   Try 6X18: $A = 93.5 \text{in}^2, \ d = 17.0 \ b = 5.5''$
5. Use larger of $L_{ex}/d = 144''/17 = 8.47, \ L_{ey}/b = 96/5.5 = 17.45$
6. $E_{min}' = 440,000(1)(1.5) = 660,000 \text{psi}$
7. $F_{CE} = 0.822(E_{min}')/(L_{ex}/d)^2 = 0.822(660,000)/17.45^2 = 1781.66 \text{psi}$
8. $F = F_{CE}/F_c^* = 1781.66/907.2 = 1.964$
9. $c = 0.8$ for sawn lumber,
10. $C_p = (1 + F)/2c - \sqrt{[(1 + F)/2c]^2 - (F/c)} = 0.864$
11. $F_c' = F_c(C_p)(C_p) = \text{allowable compressive stress} = 907.2(0.864)(12/17)^{1/9} = 754.07 \text{psi}$
12. $f_c = P/A = \text{actual compressive stress} = 90,000#/93.5 \text{in}^2 = 962.57 \text{psi}$
13. Is $f_c < F_c'$? NO. $962.57 \text{psi} > 754.07 \text{psi}$

Try 6X24: $A = 126.5 \text{in}^2, \ d = 23, \ b = 5.5''$
5. Use larger of $L_{ex}/d = 144''/23 = 6.26, \ L_{ey}/b = 96/5.5 = 17.45$
6. $E_{min}' = 440,000(1)(1.5) = 660,000 \text{psi}$
7. $F_{CE} = 0.822(E_{min}')/(L_{ex}/d)^2 = 0.822(660,000)/17.45^2 = 1781.66 \text{psi}$
8. $F = F_{CE}/F_c^* = 1781.66/907.2 = 1.964$
9. $c = 0.8$ for sawn lumber,
10. $C_p = (1 + F)/2c - \sqrt{[(1 + F)/2c]^2 - (F/c)} = 0.864$
11. $F_c' = F_c(C_p)(C_p) = \text{allowable compressive stress} = 907.2(0.864)(12/23)^{1/9} = 729.16 \text{psi}$
12. $f_c = P/A = \text{actual compressive stress} = 90,000#/126.5 \text{in}^2 = 711.46 \text{psi}$
13. Is $f_c < F_c'$? YES. $711.46 \text{psi} < 729.16 \text{psi}$

ANSWER: USE 6X24

17-5. A 6X10 column of Select Structural DFL is 20’ long with fixed ends and has a factored axial load of 50000#, a factored Mx of 400 #-in and My of 200 #-in. Is this column adequate?

1. $F_b = 1500 \text{psi}, \ F_c = 1150 \text{psi}, \ E_{min} = 580,000 \text{psi}$
2. Find section properties: $A = 50.88 \text{in}^2, \ S_x = 78.43 \text{in}^3, \ S_y = 46.64 \text{in}^3$
3. $f_{b1} = M_x/S_x = 400/78.43 = 5.10 \text{psi}, \ f_{b2} = M_y/S_y = 200/46.64 = 4.29 \text{psi}$
4. $E_{min}' = 580,000(1)(1.5) = 870,000 \text{psi}$
5. $L_{u} = 20ft(12'') = 240'' \ L_e = kL = 0.65(240'') = 156''$
6. $L_{ex}/d_1 = 156/9.25 = 16.86$
7. $L_{ex}/d_2 = 156/5.5 = 28.36$
8. $F_{CE1} = 0.822(870,000)/16.86^2 = 2515.80 \text{psi} \ > 982.7 \text{psi} = f_c \ \text{... okay}$
9. $F_{CE2} = 0.822(870,000)/28.36^2 = 889.16 \text{psi} < 982.7 \text{psi} = f_c \ \text{... NO GOOD!}$
17-6. Check the adequacy of an 8X16 timber beam with \( L = 24' \), \( Lu = 8' \), two concentrated loads of 3000# each at 8’o.c. and, a tension load of 1500#, structural select northern red oak.

\[
F_b = 1600\text{psi, } F_t = 950\text{psi, } F_v = 205\text{psi, } E = 13000000\text{psi, } E_{min} = 470000\text{psi, } G = .68
\]

8X16: \( A = 108.75, S = 271.88, I = 2039.06 \text{ C} \)

\[
F_b' = 1600(1)[1][1][\text{CL}](.98)[2.16][0.8] = 2709.50\text{C}_L
\]

\[
C_v: \frac{d}{b} = 15/7.25 = 2.07 \quad Lu = 96", \quad Lu/d = 96/15 = 6.4
\]

1. From Table 16.6, \( L_e = 1.68Lu = 1.68(96) = 161.28" \)
2. \( R_b^2 = \frac{L_e(d)}{b^2} = 161.28(15)/7.25^2 = 46.03 \)
3. Check that \( R_b^2 = 46.03 \leq 2500. \) Yes ... okay
4. \( E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5) = 470,000(1)(1)(1)(1.5) = 705,000\text{psi} \)
5. \( F_{be}' = 1.2(E_{min}')/R_b^2 = 1.2(705,000)/46.03 = 18379.32\text{psi} \)
6. \( F_b = 2709.50\text{psi} \)
7. \( F = F_{be}'/F_b^* = 18379.32/2709.50 = 6.783 \)
8. CL = \( \frac{(1 + F)}{1.9} \) – \( \sqrt{\left(\frac{(1 + F)}{1.9}\right)^2 - \left(\frac{F}{0.95}\right)} = 0.992 \)

\[
F_b^* = F_b' (C_v) = 2709.50(0.992) = 2687.82\text{psi}
\]

Factored weight of beam = 1.2[.68](62.4)[108.75/144] = 38.45#/f

Assume concentrated loads are live loads: \( P_u = 3000(1.6) = 48000# \)

\[
M = wL^2/8 + PL/3 = 38.45(24)^2/8 + 4800(24)(12)/3 = 494,020.8\#-\text{in}
\]

\[
fb = M/S = 494,020.8/271.88 = 1817.05 < 2687.82 \quad \text{okay for flexure}
\]

2. Check Tension:

\[
F_t' = 950[2.16][0.8] = 1231.2\text{psi}
\]

\[
F_t = P/A = 1500/108.75 = 13.79 < 1231.2 \quad \text{psi for tension}
\]

3. Check flexure and tension combined:

\[
Ft/Ft + fb/Fb^* = 13.79/1231.2 + 1817.05/ 2709.5 = 0.68 < 1.0 \quad \text{okay}
\]

4. Check Shear:

\[
F_v' = F_v(C_m)(C_t)[2.16]/(\lambda) = 205(1)[1][2.16][0.8] = 354.24
\]

\[
V = 38.45(24)/2 + 3000 = 3461.4#\]

\[
fv = 3V/2A = 3[3461.4/(240108.(75))] = 47.74 < 354.24 \quad \text{okay for shear}
\]

5. Check Deflection:

\[
\Delta all = L/240 = 24(12)/240 = 1.2"
\]

\[
E' = E(C_m)(C_t) = 1,300,000(1)(1) = 1,300,000 \text{ psi}
\]

Unfactored load: \( P = 3000#, \quad W = .68[62.4](5.5)(9.25)/144 = 14.99#/f \)

\[
\Delta max = 5wL^4/384El + 23PL^2/648El = 5(38.451.2)(24)^4(1728)/384(130000)/2039.06 + 3000(24)^3(1728)/648(1300000)/2039.06 = 0.13" < 1.2" \quad \text{OKAY}
Design a 10.5” wide Southern Pine 28F-2.1E SP Glu-Lam, spanning 60’ with concentrated live loads of 2000# and concentrated dead loads of 1000# spaced 10’ o.c. The beam is curved such that the midpoint of the beam is 6ft above the supports. The laminations are 0.75” thick. Blocking occurs at points of load and at ends.

1. \( F_b = 2800 \text{psi}, \quad F_v = 300 \text{psi}, \quad E = 2,100,000 \text{psi}, \)
2. Assume factored beam weight = \( W_{FBM} = 60'(10)#/f = 600#/f \)
3. \( F_{b'} = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda) \)
   \( C_{fu}: \) Are laminations vertical and depth, \( dy < 12"? \) No: \( C_{fu} = 1.0 \)
   \( C_c = 1 - 2000(t/R)^2: \) find \( R: \)
   \[ 30^2 + (R - 6)^2 = R^2 \]
   \[ R = (900 + 36)/12 = 78' = 936" \]
   \( t/R = 0.75/936 = .0008 < 1/125 = .008 \) … okay
   \( C_c = 1 - 2000(.0008) \)
   \[ 2 = 0.999 \]
   \( F_{b'} = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda) = 2800[1][1][1][.999][2.16][.8] = (F_{b*})(C_L)(C_v) = 4833.56(C_j)(C_v) \)
4. \( W_u = W_{FBM} = 600#/f \)
5. \( M_{max} = wL^2/8 + 3PL/4 = 600#/f(60')^2/8 + 3(4400#)(60')/4 = 468,000#/f = 5,616,000#-in \)
6. \( S_{req} > M/F_{b'} = 5,616,000#-in/4833.56psi = 1161.88in^3 \)
7. Try 10.5” X 26.125”: \( A = 274.3in^2, \quad S_x = 1194in^3, \quad I_x = 15600in^4 \)
8. \( d/b = 26.125/10.5 = 2.49 \) Condition states \( CL = 1 \) if \( c) 2 \leq d/b \leq 4 \) AND edges are secured by blocking or X-bracing. Therefore, \( CL = 1 \)
9. \( C_v = (21/L)^{1/X}(12/d)^{1/X}(5.125/b)^{1/X} = (21/60)^{1/20}(12/26.125)^{1/20}(5.125/10.5)^{1/20} = 0.881 \leq 1.0 \)
   \( X = 20 \) (southern pine)
10. \( F_{b'} = [F_{b*}][\text{lesser of } (C_L) \text{ or } C_v] = 4833.56(0.881) = 4256.00psi \)
11. \( W_{BM} = \text{specific gravity})(62.4pcf)(A/144)#/f = (.55)(62.4)(274.3/144) = 65.37#/f \)
12. Find Factored Loads using the six equations at the beginning of this chapter. If there are only dead and live loads:
   \( W_u = 1.2(65.37#/f) = 78.45 \)
   \( Pu = 1.2(1000#) + 1.6 (2000#) = 4400# \) & 5 point loads equally spaced.
5. \( M_{max} = wL^2/8 + 3PL/4 = 78.45#/f(60')^2/8 + 3(4400#)(60')/4 = 233302.5#-f = 2,799,630#-in \)
14. \( fb = M/S = 2,799,630/1194 = 2344.75psi \)
15. \( Is fb \leq F_{b'}? \) Yes. 2344.75psi < 4256.0psi
16. \( Is fb/F_{b'} \geq 0.90? \) No → could try smaller size, but will need depth for deflection.
17. \( Fv' = Fv(C_m)(C_t)(2.16) \quad (\lambda) = 300psi[1](1)[2.16][.8] = 518.4psi \)
18. \( V = wL/2 + 5P/2 = (78.45#/f)(60')/2 + 5(4400#)/2 = 13,353.5# \)
19. \( f_v = 78.45#/f = 3(13353.5)/[2(274.3)] = 73.02psi \)
20. Is \( f_v \leq F_{v'}? Yes. 73.02psi < 518.4psi \)
21. \( \Delta_{all} = L(12''/f)/240 = 60'(12''/f)/240 = 3'' \)
22. \( w = 65.37#/f, \quad P = 1000# + 2000# = 3000# \)
23. \( E' = E(C_m)(C_t) = 2,100,000psi(1)[1] = 2,100,000psi \)
24. \( \Delta_{act} = 5wL(384EI) + 11PL(144EI) = 5(65.37)(60')(1728)/384/2,100,000/15600 \)
           + 11(3000)(60')(1728)/144/2,100,000/15600 = 0.582 + 2.611 = 3.193''
25. Is \( \Delta_{act} \leq \Delta_{all}? No \quad \text{find } I_{req} = \Delta_{act}(lx \text{ from step 3})/\Delta_{all} = 3.193(15600)/3 = 16603.6in^4 \)
USE 10.5 X 27.5: \( I = 18200in^4 \)
18-2. Design a 10.5" wide, 48/SP/N2D12 column, 30ft long, to carry a factored axial load of 40,000# with pinned connections and a moisture content of 20% and an average temperature to 105°.

1. $F_c = 2200\text{psi}$ and $E_{in} = 900,000\text{psi}$.
2. $F_c' = F_c(C_m)(C_t)(1.5) = 2200(0.8)(1)(1.5) = 3041.28\text{(C_p)}$
3. $L_e = kL(12''/f) = 1(30')(12''/f) = 360''$
4. $d_{in} = b_{in} = L_e/50 = 360/50 = 7.2''$
5. $A_{trial} = P/F_c* = 40,000#/3041.28 = 13.15$ Try 10.5" X 11: $A = 115.5, b = 10.5, d = 11$
6. $E_{in}' = E_{in}(C_t)(C_m)(1.5) = 900,000\text{psi}(1)(0.8)(1.5) = 1,080,000\text{psi}$
7. $F_{ce} = 0.822(E_{in}')/(L_e/d)^2 = 0.822(1,080,000)/34.92^2 = 728.03\text{psi}$
8. $F = F_{ce}/F_c* = 728.03/3041.28 = 0.24$
9. $c = 0.9$ for glu-lams
10. $C_p = (1 + F)/2c - (((1 + F)/2c)^2 - (F/c))^{1/2} = (1.24)/1.8 - (((1.24)/1.8)^2 - (.24/.9))^{1/2} = 0.233$
11. $F_c' = F_c(C_p) = 3401.28(0.233) = 792.25\text{psi}$
12. $f_c = P/A = 40,000/115.5 = 346.32\text{psi}$
13. Is $f_c < F_c'$? Yes → $346.32 < 792.25\text{psi}$
14. Is $f_c/F_c' ≥ 0.90$? No, $346.32/792.25 = .44$ but 10.5 X 11 is the smallest available 10.5" wide size.
ANSWER: USE 10.5" X 11"

20-1. Find the most economical W14 for an A992 steel beam spanning 40' with a dead load of 50psf and a live load of 80psf if the beams are spaced at 12' o.c. and full lateral bracing is provided.

1. $W_u = 1.2WD + 1.6WL = 1.2(50\text{psf})(12') + 1.6(80\text{psf})(12') = 2256#/f = 2.26\text{k/f}$
2. $M_u = wL^2/8 = 2.26\text{k/f}(40')^2/8 = 5424\text{in}$
3. $Z_{req'd} = M_u/0.9F_y = 5424\text{in}/[0.9(50\text{ksi})] = 120.53\text{in}^3$

Try a W14X74: $Z_x = 126\text{in}^3, I_x = 795\text{in}^4, tw=0.45''$, $d = 14.2''$
4. $M_{u_{BM}} = wL^2/8 = 1.2(74/1000)(40)^2(12''/f)/8 = 213.12\text{k-in}$
New $M_u = M_{u_{BM}} + M_{u_{BM}} = 5424\text{k-in} + 213.12\text{k-in} = 5637.12\text{k-in}$.
5. full lateral bracing. Therefore ZONE 1: $Lb ≤ Lp$
6. $ΦM_n = ΦM_p = 0.9F_yZ = 0.9(50\text{ksi})(126\text{in}^3) = 5670\text{k-in}$
7. $ΦM_n = 5670\text{k-in} > M_u = 5637.12\text{k-in}$, therefore, OKAY.
8. Check Shear: $V_u = (2.26\text{k/f} + 1.2(0.074\text{k/f}))(40')/2 = 46.98\text{k}$
9. $ΦV_{nx} = 0.6F_y(twd) = 0.6(50)(0.45)(14.2) = 191.7k$
$ΦV_{nx} = 191.7k > V_u = 46.98$, therefore the beam is OKAY for shear.
10. Check deflection: $Δ_{all} = 40'(12''/f)/240 = 2.0''$
11. $w = [(50 + 80\text{psf})(12') + 74\text{#/f}]/1000#/k = 1.63\text{k/f}$
$Δ_{actual} = 5wL^4/[384EI] = 5(1.63)(40')^4(1728)/384/29000/795 = 4.07''$
$Δ_{all} = 2.0'' < Δ_{actual} = 4.07''$
Ixnew = [Ixused]/[Δ_{actual}]/[Δ_{all}] = (795\text{in}^4)(4.07'')/2'' = 1617.83\text{in}^4$
USE: W14X145, $I_x = 1710$
20-2. Find the most economical W14 for an A992 steel beam spanning 35' with concentrated dead loads of 1k and concentrated live loads of 2k spaced at 5' o.c. if lateral bracing is only provided at the point loads.

1. \( P_u = 1.2PD + 1.6PL = 1.2(1k) + 1.6(2k) = 4.4k \) 7spaces = 6 point loads
2. \( M_u = 6PL/7 = 6(4.4k)(35')(12\text{in/f})/7 = 1584\text{k-in} \)
3. \( Z_{\text{req'd}} = M_u/0.9F_y = 1584\text{k-in}/(0.9[50\text{ksi}]) = 35.2\text{in}^3 \)
   - Try a W14X26: \( Z = 40.2\text{in}^3, S_x = 35.3 \text{in}^3, I_x = 245\text{in}^4, r_y = 1.08", t_w = 0.255", d = 13.9" \)
4. \( M_{u_{\text{BM}}} = wL^2/8 = 1.2(.026\text{k/f})(35')(12\text{in/f})/8 = 57.33\text{k-in} \)
   - New \( M_u = 1584 + 57.33 = 1641.33 \text{k-in} \)
5. \( L_b = 5' \) as stated in the problem.
   - From Table 20.1: \( L_p = 3.81', L_r = 11.1' \) ...
   - ZONE 2: \( L_p \leq L_b \leq L_r \)
   - \( \Phi_{bM_p} = 0.9F_y Z = 0.9[50\text{ksi}](40.2\text{in}^3) = 1809\text{k-in} \)
6. \( \text{ZONE 2: } \Phi_{bM_n} = C_b[\Phi_{bM_p} - (\Phi_{bM_p} - F_y S_x)(L_b - L_p)] \leq \Phi_{bM_p} \)
   - \( C_b = 1.0 \)
   - \( \Phi_{bM_n} = 1.0[1809\text{k-in} - (1809 - 50[35.3])(5.0 - 3.81)] = 2100.35\text{k-in} > 1809 \) ...
   - \( \Phi_{bM_n} = 1809\text{k-in} \).
7. \( \Phi_{bM_n} = 1809 > M_u = 1641.33 \text{k-in}, \) therefore, OKAY for flexure
8. Check Shear: \( V_u = 6(4.4)/2k + 1.2(.026k/f)(35')/2 = 25.95k \)
9. \( \Phi_{V_{nx}} = 0.6F_y(t_wd) = 0.6(50)[0.255](13.9) = 106.34k \)
   - \( \Phi_{V_{nx}} = 106.34k > V_u = 25.95, \) therefore the beam is OKAY for shear.
10. Check deflection: Allowable deflection = \( L(12\text{in/f})/240 = \Delta_{\text{all}} = 35'(12\text{in/f})/240 = 1.75" \)
   - \( P = 1k + 2k = 3k, \ w = .026k/f \)
   - \( \Delta_{\text{actual}} = 123PL^3/[1728]/[1372\text{EI}] + 5wL^4/[384\text{EI}] = 123(3k)(35')^3(1728)/1372/29000/245 + 5(0.025)(35')(1728)/384/29000/245 = 2.92 >1.75" \) No Good
   - \( I_{\text{new}} = [I_{\text{used}}][\Delta_{\text{actual}}]/[\Delta_{\text{all}}] = (245\text{in}^4)(2.92")/1.75" = 408.8\text{in}^4 \)
   - USE: W14X43: \( I_x = 428\text{in}^4 \)
20-3. Find the most economical W14 for an A992 steel beam spanning 30’ with concentrated dead loads of 10k and concentrated live loads of 20k at midspan if
a) No lateral bracing is provided.

1. \( Pu = 1.2PD + 1.6PL = 1.2(10k) + 1.6(20k) = 44k \) @ center
2. \( Mu = PL/4 = 44k(30')/12in/f)/4 = 3960k-in \)
3. \( Z_{req'd} = Mu/0.9Fy = 3960k-in/[0.9(50ksi)] = 88in^3 \)
   
   Try a W14X61: \( Z = 102in^3, S_x = 92.1in^3, I = 640in^4, r_y = 2.78", tw = 0.375", d = 13.9" \)

4. \( Mu_{bm} = wL^2/8 = 1.2(0.061)(30')^2/8 = 98.82k-in \)

   New Mu = 3960k-in + 98.82k-in = 4058.82k-in.

5. \( Lb = 30' \) as stated in the problem. \( Lp = 8.65', Lr = 27.5' \) ... ZONE 3: \( Lr \leq Lb \)
   
   NOTE: I recommend telling students to avoid Zone 3 by choosing a larger size. A W14X74 would put them into Zone 2, for example with only 13#/ft additional weight, but this is a case where it works.

6. ZONE 3: \( FbMn = 0.9FcSx \) and \( Fcr = [C_b\pi^2E/(L_b/\sqrt{3})]^2 \sqrt{1 + 0.078(Jc/Sxho)(L_b/\sqrt{3})^2} \)

   From Figure 20.3 : \( C_b = 1.32. \)

   From Table 1-1 of the AISC Steel Manual, the following values are obtained:

   \( r_s = 2.78, J = 2.19, S_x = 92.1, ho = 13.2 \) and \( c = 1.0 \) because W shapes are doubly-symmetrical.

   \( (L_b/r_s)^2 = ((30')(12'/f)/2.78)^2 = 16,769.32 \)

   \( Jc/S_xho = 2.19/92.1/13.2 = .0081 \)

   \( Fcr = [1.0(3.14159)^2(29000)/ (16769.32)]^2 \sqrt{1 + 0.078(.0081)(16769.32)} = 58.19ksi \)

   \( FbMn = 0.9FcSx = 0.9(58.19ksi)(92.1in^3) = 4823.37k-in \)

   \( FbMp = 0.9FyZ = 0.9(50ksi)(102in^3) = 4590k-in \) USE THE LESSER VALUE

7. \( FbMn = 4590k-in > Mu = 4058.82k-in, \) therefore, the beam OKAY for flexure!

8. Check Shear: \( Vu = 44/2k + 1.2(0.061k/f)(30')/2 = 23.098k \)

9. \( \Phi Vn = 0.6Fy(twd) = 0.6(50)(0.375)(13.9) = 156.38k \)

   \( \Phi Vnx = 156.38k > Vu = 23.10k, \) therefore the beam is okay for shear.

10. Check deflection: Allowable deflection = \( L(12''/f)/240 = \Delta all = 30'(12''/f)/240 = 1.5'' \)

    \( P = 10k + 20k = 30k, \ w = .061k/f \)

    \( \Delta actual = PL^3(1728)/[48EI] + 5wL^4(1728)/[384EI] = (30k)(30')(1728)/48/29000/640 \)

    \( + 5(0.061)(30')(1728)/384/29000/640 = 1.63'' \)

11. \( \Delta all = 1.5'' < \Delta actual = 1.63'' \)

12. \( Ixnew = [Ixused] [\Delta actual]/[\Delta all] = (640in^4)(1.63'')/1.5'' = 695.47in^4 \)

   USE: W14X68, \( Ix = 722 \)

b) Lateral bracing is provided at midspan

5. \( Lb = 15' \) as stated in the problem. \( Lp = 8.65', Lr = 27.5' \) ... ZONE 2:

6. ZONE 2: \( \Phi bMn = C_b[\Phi bMp – (\Phi bMp – FySx)(Lb – Lp)] \leq \Phi bMp \)

   \( C_b = 1.67 \)

   \( \Phi bMp = 0.9FyZ = 0.9(50ksi)(102in^3) = 4590k-in \)

   \( \Phi bMn = 1.67[4590 – 1.67(4590 – 50(92.1))(15.0 – 8.65)] = 7930.94k-in > 4590 \)

   \( \Phi bMn = 4590k-in. \)

and the remainder of the problem is the same as above.
21-1: Design most economical W14, L = 30', PD = 200k, PL = 400k using A992 steel for the following end conditions:
   a) Both ends fixed
   b) One end fixed and one end pinned
   c) Both ends pinned

1. Pu = 1.2(200) + 1.6(400) = 880k
2. Assume kL/r = 50
3. Find \( \Phi_{cFcr} \)
   - E3-4: \( Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(50)^2 = 114.49 \text{ksi} \)
   - 4.71 \( \sqrt{(E/Fy)} = 4.71 \sqrt{(29,000/50)} = 113.43 \)
   - \( E3-2: kL/r < 4.71 \sqrt{(E/Fy)} \), therefore \( \Phi Fcr = 0.658(\sqrt{E/Fy})Fy = 0.658(50)(50) = 37.5 \text{ksi} \)
4. \( Atrial = Pu/\Phi_{cFcr} = 880k/37.5k = 23.47 \text{in}^2 \)
5. Try W14X82: \( A = 24 \text{in}^2 \), \( ry = 2.48" \)
   a) Both ends fixed ... \( k = 0.65 \)
6. \( kL/r = 0.65(30)/(12) = 0.65 \times 2.5 = 94.35 \)
7. Find actual \( \Phi_{cFcr} \):
   - E3-4: \( Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(94.35)^2 = 32.15 \text{ksi} \)
   - 4.71 \( \sqrt{(E/Fy)} = 4.71 \sqrt{(29,000/50)} = 113.43 \)
   - \( E3-2: kL/r < 4.71 \sqrt{(E/Fy)} \), therefore \( \Phi Fcr = 0.658(\sqrt{E/Fy})Fy = 0.658(50)(73.91) = 30.24 \text{ksi} \)
8. \( fc = P/A = 880k/24\text{in}^2 = 36.67 \text{ksi} \)
9. \( \Phi_{cFcr} = 23.48 < fc = 36.67 \) therefore the column is not adequate. Go back to step 5 and try larger size. \( Atrial = 24(36.67)/23.48 = 37.48 \text{in}^2 \)
5A. Try W14X132: \( A = 38.3, ry = 3.76 \)
6A. \( kL/r = 0.65(30)/(12) = 0.65 \times 2.5 = 94.35 \)
7A. Find actual \( \Phi_{cFcr} \):
   - E3-4: \( Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(94.35)^2 = 32.15 \text{ksi} \)
   - 4.71 \( \sqrt{(E/Fy)} = 4.71 \sqrt{(29,000/50)} = 113.43 \)
   - \( E3-2: kL/r < 4.71 \sqrt{(E/Fy)} \), therefore \( \Phi Fcr = 0.658(\sqrt{E/Fy})Fy = 0.658(50)(73.91) = 30.19 \text{ksi} \)
8A. Calculate the actual compressive stress = \( fc = P/A = 880k/38.3\text{in}^2 = 22.98 \text{ksi} \)
9A. \( \Phi_{cFcr} = 30.19 > fc = 22.98 \) therefore the column is adequate.
10. \( fc/\Phi_{cFcr} = 22.98/30.19 = 0.76 < 0.90 \), try a smaller size.
5B. Try W14X99: \( A = 29.1, ry = 3.71 \)
6B. \( kL/r = 0.65(30)/(12) = 0.65 \times 2.5 = 94.35 \)
7B. Find actual \( \Phi_{cFcr} \):
   - E3-4: \( Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(62.23)^2 = 73.91 \text{ksi} \)
   - 4.71 \( \sqrt{(E/Fy)} = 4.71 \sqrt{(29,000/50)} = 113.43 \)
   - \( E3-2: kL/r < 4.71 \sqrt{(E/Fy)} \), therefore \( \Phi Fcr = 0.658(\sqrt{E/Fy})Fy = 0.658(50)(71.95) = 33.65 \text{ksi} \)
8B. Calculate the actual compressive stress = \( fc = P/A = 880k/29.1\text{in}^2 = 30.24 \text{ksi} \)
9B. \( \Phi_{cFcr} = 33.65 > fc = 30.24 \) therefore the column is adequate.
10B. \( fc/\Phi_{cFcr} = 30.24/33.65 = 0.90 ... \) ADEQUATE
b) One end pinned, one end fixed ... k = 0.865
Try W14X99: A = 29.1, ry = 3.71
6. kL/r = 0.8(30)(12)/3.71 = 77.63
7. Find actual$\Phi cF_{cr}$:
   E3-4: $Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(77.63)^2 = 47.49$ksi, $4.71\sqrt{(E/Fy)} = 113.43$
   E3-2: $kL/r \leq 4.71\sqrt{(E/Fy)}$, therefore $\Phi F_{cr} = .9(658^{(E/Fy)}Fy = .9(658^{(50/47.49)})(50) = 28.97$ksi
8. $fc = P/A = 880k/29.1in^2 = 30.24$ksi
9. $\Phi cF_{cr} = 28.97 < fc = 30.24$ therefore GO LARGER ... $A = 29.1(30.24/28.97) = 30.38$

TRY W14X109: A = 32, ry = 3.73
6A. kL/r = 0.8(30)(12)/3.73 = 77.21
7A. Find actual$\Phi cF_{cr}$:
   E3-4: $Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(77.21)^2 = 48.01$ksi, $4.71\sqrt{(E/Fy)} = 113.43$
   E3-2: $kL/r \leq 4.71\sqrt{(E/Fy)}$, therefore $\Phi F_{cr} = .9(658^{(E/Fy)}Fy = .9(658^{(50/48.01)})(50) = 29.11$ksi
8A. $fc = P/A = 880k/32in^2 = 27.5$ksi
9A. $\Phi cF_{cr} = 29.11 > fc = 27.5$ therefore adequate
10. $fc/\Phi cF_{cr} = 27.5/29.11 = .94...$ efficient
USE W14X109

c) Both ends pinned, one end fixed ... k = 1.0

TRY W14X109: A = 32, ry = 3.73
6. kL/r = 1.0(30)(12)/3.73= 96.51
7. Find actual$\Phi cF_{cr}$:
   E3-4: $Fe = \pi^2E/(kL/r)^2 = \pi^2(29,000)/(96.51)^2 = 30.73$ksi, $4.71\sqrt{(E/Fy)} = 113.43$
   E3-2: $kL/r \leq 4.71\sqrt{(E/Fy)}$, therefore $\Phi F_{cr} = .9(658^{(E/Fy)}Fy = .9(658^{(50/30.73)})(50) = 22.78$ksi
8. $fc = P/A = 880k/32in^2 = 27.5$ksi
9. $\Phi cF_{cr} = 22.78 < fc = 27.5$ therefore adequate
10. $fc/\Phi cF_{cr} = 27.5/22.78 = .94...$ efficient
USE W14X145
21-2: Find $\Phi cPn$ of 18ft HSS 10X8X1/2 fixed at both ends and $F_y = 46$ksi.

1. $A = 15.3$, $rx = 3.73$, $ry = 3.14$, $t = 0.465$, $b/t = 14.2$, $h/t = 18.5$
2. $be_b = 1.92t\sqrt{[E/F_y][1 - (0.38/(b/t))\sqrt{E/F_y}] = 1.92(0.465)\sqrt{29000/46}[1 - (0.38/(14.2))\sqrt{29000/46}] = 7.35$
3. $b = B - 3t = 8 - 3(0.465) = 6.605''$
4. $be_h = 1.92t\sqrt{[E/F_y][1 - (0.38/(h/t))\sqrt{E/F_y}] = 1.92(0.465)\sqrt{29000/46}[1 - (0.38/(18.5))\sqrt{29000/46}] = 10.86$
5. $h = H - 3t = 10 - 3(0.465) = 8.605$
6. $Ae = A - 2(t)(h - be_h) - 2(t)(b - be_b)$
   $b - be_b = 6.605 - 7.35 = -0.745$
   $h - be_h = 8.605 - 10.86 = -2.255$
   $Ae = A - 2t(h - be_h) - 2t(b - be_b) = 15.3 - 2(0.465)(-2.255) - 2(0.465)(-0.745) = 18.0$
7. $Q = Ae/A = 18/15.3 = 1.176$ ... member is not slender
8. $kL/r = 0.65(18)(12)/3.14 = 44.71$
9. $Fe = \pi^2E/(kL/r)^2 = \pi^2E/(kL/r)^2 = 143.16$
10. Find $F_{cr}$: $4.71\sqrt{E/F_y} = 4.71\sqrt{29000/46} = 118.26 > kL/r$ therefore $F_{cr} = [0.658\sqrt{F_y/F_e}]F_y = [0.658(46/143.16)]46 = 40.21$ksi
   $\Phi cFcr = 0.9F_{cr} = 0.9(40.21) = 36.19$ksi
11. $cPn = \Phi cFcrAg = 36.19(15.3) = 553.71$k
21-3: Find the maximum compressive load, $\Phi_{Pn}$ for a pinned W14X120 with a 1/2" X 12" plate bolted to each flange with a bolt spacing of 12" along the length of the 24' column. $F_y = 50$ksi.

1. W14X120  $A = 35.3\text{in}^2$, $d = 14.5\text{"}$, $r_x = 6.24\text{"}$, $r_y = 3.74\text{"}$
   PL1/2X12:  $A = 6$, $I_x = 12(0.5^3)/12 = 0.125\text{in}^4$, $I_y = .5(12^3)/12 = 72\text{in}^4$
   \[ r_{xi} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{0.125}{6}} = 0.144\text{"} \]
   \[ r_{yi} = \sqrt{\frac{I_y}{6}} = 3.464\text{"} \]
   $a =$ spacing of bolts = 12"
   $d_y = 14.5/2 + .5/2 = 7.5$, $dx = 0$

   Built-up properties:
   \[
   \begin{array}{c|c|c|c|c}
   \text{Comp.} & A_i & I_{xi} & d_y & A_dy^2 \\
   \hline
   \text{W14X120} & 35.3 & 1380 & 0 & 0 \\
   .5X12 & 6 & 0.125 & 7.5 & 337.5 \\
   .5X12 & 6 & 0.125 & 7.5 & 337.5 \\
   \hline
   \sum A_i = 47.3 & \sum I_{xi} = 1380.25 & \sum A_dy^2 = 675
   \end{array}
   \]

   \[ I_x = \sum I_{xi} + \sum A_dy^2 = 1380.25 + 675 = 2055.25\text{in}^4 \]
   \[ r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2055.25}{47.3}} = 6.592\text{"} \]

   \[
   \begin{array}{c|c|c|c|c}
   \text{Comp.} & A_i & I_{yi} & d_x & A_{dx^2} \\
   \hline
   \text{W14X120} & 35.3 & 495 & 0 & 0 \\
   .5X12 & 6 & 72 & 0 & 0 \\
   .5X12 & 6 & 72 & 0 & 0 \\
   \hline
   \sum A_i = 47.3 & \sum I_{yi} = 639 & \sum A_{dx^2} = 0
   \end{array}
   \]

   \[ I_y = \sum I_{yi} + \sum A_{dx^2} = 639\text{in}^4 \]
   \[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{639}{47.3}} = 3.676\text{"} \]

2. Modified slenderness ratio:
   \[ kL/rmx = \sqrt{\left(\frac{kL}{r} \right)^2 + \left(\frac{a}{r_i} \right)^2} = \sqrt{\left(\frac{1(24(12\text{in/f})/6.592\text{")})^2}{6.592}\right)^2 + \left(\frac{12/1.144}{1.144}\right)^2} = 94.091 \]
   \[ kL/rmy = \sqrt{\left(\frac{kL}{r} \right)^2 + \left(\frac{a}{r_i} \right)^2} = \sqrt{\left(\frac{1(24(12\text{in/f})/3.676\text{")})^2}{12/3.464}\right)^2 + \left(\frac{12/3.464}{3.464}\right)^2} = 78.423 \]

4. Find $\Phi_{cFcr}$:
   \[ Fe = \pi^2E/(kL/r)^2 = \pi^2(29000\text{ksi})/(94.091)^2 = 32.330\text{ksi} \]
   \[ 4.71\sqrt{E/Fy} = 4.71\sqrt{29000/50} = 113.43 >94.091 \]
   \[ F_{cr} = (.658\text{Fy/Fe})F_y = (.658(50/32.33))(50) = 26.173\text{ksi} \]
   \[ \Phi_{cFcr} = .9(26.173) = 23.56\text{ksi} \]
   \[ \Phi_{cPn} = \Phi_{cFcr}A = 23.56(47.3) = 1114.17\text{k} \]
21-4: Repeat exercise 21-3 if the bolts at 12” o.c. are replaced with 2”welds at 18”o.c.

1. W14X120  \( A = 35.3 \text{in}^2, d = 14.5”, r_x = 6.24”, r_y = 3.74” \)
   PL1/2X12:  \( A = 6, l_x = 12(0.5^3)/12 = 0.125 \text{in}^4, l_y = 0.5(12^3)/12 = 72 \text{in}^4 \)
   \( r_x = \sqrt{l_x/A} = \sqrt{0.125/6} = 0.144” \)
   \( r_y = \sqrt{l_y/6} = 3.464” \)
   \( a = \text{spacing of bolts} = 12” \)
   \( d_y = 14.5/2 + 0.5/2 = 7.5, d_x = 0 \)

   Built-up properties:

<table>
<thead>
<tr>
<th>Comp.</th>
<th>( A_i )</th>
<th>( l_xi )</th>
<th>( d_y )</th>
<th>( A_dy^2 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>35.3</td>
<td>1380</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.5X12</td>
<td>6</td>
<td>0.125</td>
<td>7.5</td>
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<td>7.5</td>
<td>337.5</td>
</tr>
<tr>
<td>( \sum A_i = 47.3 )</td>
<td>( \sum l_xi = 1380.25 )</td>
<td>( \sum A_dy^2 = 675 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   \( I_x = \sum l_xi + \sum A_dy^2 = 1380.25 + 675 = 2055.25 \text{in}^4 \)
   \( r_x = \sqrt{I_x/A} = \sqrt{2055.25/47.3} = 6.592” \)

<table>
<thead>
<tr>
<th>Comp.</th>
<th>( A_i )</th>
<th>( l_yi )</th>
<th>( d_x )</th>
<th>( A_dx^2 )</th>
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<td>( \sum A_i = 47.3 )</td>
<td>( \sum l_yi = 639 )</td>
<td>( \sum A_dx^2 = 0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   \( I_y = \sum l_yi + \sum A_dx^2 = 639 \text{in}^4 \)
   \( r_y = \sqrt{l_y/A} = \sqrt{639/47.3} = 3.676” \)

   \( \alpha = h/2\text{rib} = (14.5+.5)/2 = 7.5 \)

2. Find modified slenderness ratio:
   \( kL/r_x = \sqrt{(kL/r)^2 + 0.82(\alpha^2/(1 + \alpha^2))(\alpha/\text{rib})^2} = \sqrt{(12(24)/6.592)^2 + 0.82(7.5^2/(1 + 7.5^2))(18-2)/.144)^2} \)
   = 108.88
   \( kL/r_y = \sqrt{(kL/r)^2 + 0.82(\alpha^2/(1 + \alpha^2))(\alpha/\text{rib})^2} = (12/24/3.676) = 78.346 \)

   Find \( \Phi_{Cfcr} \):
   \( Fe = \pi^2E/(kL/r)^2 = \pi^2(290000 \text{ksi})/(108.88)^2 = 24.14 \text{ksi} \)
   \( 4.71\sqrt{E/Fy} = 4.71\sqrt{290000/50} = 113.43 \)
   \( Fc_r = (.658Fy/Fe)Fy = (.658^{0.24.14})(50) = 21.01 \text{ksi} \)
   \( \Phi_{Cfcr} = .9(21.01) = 18.91 \text{ksi} \)
   \( \Phi_{Cp_n} = \Phi_{Cfcr} A = 18.91(47.3) = 894.49 \text{k} \)
Design 2-12" channels used to form a 12" square with $F_y = 50$ksi, $P_u = 400$k, and $L = 16'$. Assume single lacing at 45° with bolts at 1" from inside edge.

1. Assume $kL/r = 50$ which means $\Phi c F_c = 37.5$ksi

2. Atrial = $400/37.5 = 10.67in^2$ total area. $A = 10.67/2 = 5.33in^2$ for each channel

3. Try C12X20.7: $A = 6.08in^2$, $I_x = 129in^4$, $I_y = 3.86in^4$, $x = .698"$, $d = 12"

4. Consider column as a whole and find $I$, $r$ and $kL/r$ values.

\[A = 6.08(2) = 12.16in^2\]
\[I_x = 258in^4\] and \[I_y = 349.55in^4\]
\[r_x = \sqrt{I_x/A} = \sqrt{258/12.16} = 4.606"
\[r_y = \sqrt{I_y/A} = \sqrt{349.55/12.16} = 5.362"
\[kL/r = 1.0(16)(12)/4.06 = 47.29\]

5. Find $\Phi c F_{cr}$:

\[F_e = \pi^2E/(kL/r)^2 = \pi^2(29000ks)/(47.29)^2 = 127.985ksi\]
\[4.71\sqrt{E/F_y} = 4.71\sqrt{2900/50} = 113.43\]
\[F_{cr} = (0.658F_y/F_e)F_y = (0.658(50)/127.985)(50) = 42.458ksi\]
\[\Phi c F_{cr} = .9(42.458) = 38.212\]

6. $f_c = P/A = 400k/12.16in^2 = 32.89ksi < \Phi c F_{cr} = 38.212$ therefore okay.

7. Consider individual C12X20.7 as columns to find allowable unbraced length and find $kL/r$

C12X20.7: $A = 6.08$, $r_x = 4.61$, $r_y = 0.797$

8. Find $\Phi c F_{cr}$:

\[F_e = \pi^2E/(kL/r)^2 = \pi^2(29000ks)/(20.06)^2 = 711.27ksi\]
\[4.71\sqrt{E/F_y} = 4.71\sqrt{2900/50} = 113.43\]
\[F_{cr} = (0.658F_y/F_e)F_y = (0.658(50)/711.27)(50) = 48.55ksi\]
\[A = 6.08in^2\] and $P = 400k/2 = 200k$ on each channel section.

\[f_c = P/A = 200/6.08 = 32.89ksi \leq 48.55ksi \ldots \text{okay.}\]
21-6: Design a 16X16" column made of 4 angles to support a factored load, \( P_u = 500k \) if \( L = 14' \). Assume double lacing at 45° with bolts 1.5" from inside edges.

1. Assume \( kL/r = 50 \) which means \( \Phi cFc = 37.5ksi \)
2. Atrial = \( 500/37.5 = 13.33in^2 \) total area. \( A = 13.33/4 = 3.33in^2 \) for each channel
   
   Try \( L4X4X1/2: \ A = 3.75, lx = ly = 5.52, rx = ry = 1.21, y = x = 1.18 \)
3. Consider column as a whole and find \( I, r \) and \( kL/r \) values.
   \( A = 4(3.75) = 15.0 \)
   \( Ix = ly = 4(5.52) + 15(8 – 1.18)^2 = 719.766in^4 \)
   \( r = \sqrt{(I/A) = \sqrt{(719.766/15)} = 6.93" \)
   \( kL/r = 1.0(14)(12)/6.93 = 24.24 \)
4. Find \( \Phi cFcr \) :
   \( Fe = \pi^2E/(kL/r)^2 = 87^2(29000ksi)/(24.24)^2 = 487.12ksi \)
   \( 4.71\sqrt{E/Fy} = 4.71\sqrt{(29000/36)} = 133.68 \) (angles are A36 steel unless otherwise noted ... \( Fy = 36ksi \))
   \( Fcr = (.658Fy/Fe)Fy = (.658^{36/487.12})^{36} = 34.90ksi \)
   \( \Phi cFcr = .9(34.90) = 31.4 \)
5. \( fc = P/A = 500k/15in^2 = 33.33ksi \) > \( \Phi cFcr = 31.4 \) therefore go larger.
   
   Try \( L4X4X5/8: \ A = 4.61, lx = ly = 6.62, rx = ry = 1.20, y = x = 1.22 \)
3. \( A = 4(4.61) = 18.44 \)
   \( Ix = ly = 4(6.62) + 18.44(8 – 1.22)^2 = 874.14in^4 \)
   \( r = \sqrt{(I/A) = \sqrt{(874.14/18.44)} = 6.89" \)
   \( kL/r = 1.0(14)(12)/6.89 = 24.38 \)
4. \( Fe = \pi^2E/(kL/r)^2 = 87^2(29000ksi)/(24.38)^2 = 481.54ksi \)
   \( 4.71\sqrt{E/Fy} = 4.71\sqrt{(29000/36)} = 133.68 \)
   \( Fcr = (.658Fy/Fe)Fy = (.658^{36/481.54})^{36} = 34.89ksi \)
   \( \Phi cFcr = .9(34.89) = 31.4 \)
5. \( fc = P/A = 500k/18.44in^2 = 27.11ksi \) < \( \Phi cFcr = 31.4 \) okay
6. no need to check efficiency because next smaller size doesn’t work.
7. Consider individual \( L4X4X5/8 \) as columns to find allowable unbraced length and find \( kL/r \).
   \( L4X4X5/8 \ A = 4.61, r = 1.20 \)
   single lacing at 45° with bolt holes 1.0" from inside edge of angles.
   \( L = 2(built-up column width – 2(bf – 1.5")) = 2(16 – 2(4 – 1.5)) = 22" \)
   \( L/r = 22/1.2 = 18.33 < 75 \) ... use EQ 5-3: \( kl/r = 60+.8(22)/1.2 = 74.67 \)
8. Find \( \Phi cFcr \) :
   \( Fe = \pi^2E/(kL/r)^2 = 87^2(29000ksi)/(74.67)^2 = 51.33ksi \)
   \( 4.71\sqrt{E/Fy} = 4.71\sqrt{(29000/36)} = 133.68 \)
   \( Fcr = (.658Fy/Fe)Fy = (.658^{36/51.33})^{36} = 26.84ksi \)
   \( \Phi cFcr = .9(26.84) = 24.16 \)
A = 4.61in² and \( P = 500k/4 = 125k \) on each angle section.
   \( fc = P/A = 125/4.61 = 27.11ksi \leq 24.16ksi \) ... okay.
Find the design strength of the connection shown. Note: W14X43, 1" bolt hole diameter.

\[ Ag_{pl} = .5(8) = 4.0 \]
\[ tf_{pl} = 0.5" \]
\[ An = 4-2(.5)(1) = 3.0 \]
\[ U=1 \]
\[ Ae = 3(1) = 3.0 \]
\[ Ag_{w} = 12.6 \]
\[ tf_{w} = 0.53" \]
\[ An = 12.6 - 2(.53)(1) = 11.54 \]
\[ 2d/3 = 2(13.7)/3 = 9.13 > 8 = bf \quad U = 0.85 \]
\[ Ae = .85(11.54) = 9.81 \]

Gross yielding:
\[ .9FyAg = .9(36)(4) = 129.6k \]
\[ 129.6k governs \]

Tensile Rupture:
\[ .75FuAe = .75(58)(3) = 130.5k \]
\[ Agv = 2(11)(.5) = 11 \]
\[ Anv = 11-2(2.5)(1)(.5) = 8.5 \]
\[ Ant = 2(((8-4)/2)-.5)(.5) = 1.5 \]
\[ .9FyAg = .9(50)(12.6) = 567k \]
\[ .75FuAe = .75(65)(9.81) = 478.24k \]
\[ Agv = 2(11)(.53) = 11.66 \]
\[ Anv = 11.66-2(2.5)(1)(.53) = 9.01 \]
\[ Ant = 2(((8-4)/2)-.5)(.53) = 1.59 \]

\[ \Phi Rn_{pl} = .75[.6(36)(11)+58(1.5)] = 243.45k \]
\[ \Phi Rn_{pl} = .75[.6(58)(8.5)+58(1.5)] = 287.1k \]
\[ \Phi Rn_{w} = .75[.6(50)(11.66)+65(1.59)] = 339.86k \]
\[ \Phi Rn_{w} = .75[.6(65)(9.01)+65(1.59)] = 341.06k \]

129.6k governs
\[ \Phi Pn = 129.6k \]
Find the design strength of the connections shown. Note: 2 lines of 1" bolt holes, 4" apart.

3 - PL 3/8 X 8  

\[ t_f = 3(3/8) = 1.125" \]  

\[ t_f = 2(1/2) = 1" \]

USE \( t = 1" \)

\[ A_{pl} = 1(8) = 8.0 \]

\[ t_{f_pl} = 1.0" \]

\[ A_n = 8 - 2(1)(1) = 6.0 \]

\[ U=1 \]

\[ A_e = 6(1) = 6.0 \]

Gross yielding:

\[ 0.9F_{yAg} = 0.9(36)(8) = 259.2k \]

Tensile Rupture:

\[ 75F_{uAe} = 0.75(58)(6) = 261.0k \]

\[ A_{gv} = 2(11)(1) = 22 \]

\[ A_{nv} = 22 - 2(2.5)(1)(1) = 17 \]

\[ A_{nt} = 2(((8 - 4)/2) - .5)(1) = 3 \]

\[ \Phi R_{n1_{pl}} = 0.75[0.6(36)(22)+58(3)] = 486.9k \]

\[ \Phi R_{n1_{pl}} = 0.75[0.6(58)(17)+58(3)] = 574.2k \]

259.2k governs

\[ \Phi P_n = 259.2k \]
22-3. Find the narrowest 6" plate thickness, \( t \), for the connection shown if \( P_u = 500k \). Use 7/8" bolts.

1. \( P_u = 500k \)ips
2. Gross Yielding: \( A_g > \frac{P_u}{(0.9(F_y))} = \frac{500k}{(0.9(36ksi))} = 15.43in^2 \)
\[ t \geq \frac{15.43}{6} = 2.57" \text{ round up to next 1/8"}, \ 2.625" \]
3. \( U = 1 \) ... \( A_e = A_n \)
4. \( r \) Cannot be determined because no length is listed.
5. Through one hole: \( A_n = (6-1)t = 5t \)
   Through two holes: \( A_n = \frac{(6-1+2^2/(4(2))]}{t} = 5.5t \)
   \( A_n = A_e = 5t \)
   \( A_e = \frac{500}{(0.75(58)(1)} = 11.49 = 5t... \ t = 2.30" < 2.625, \ ... 2.625 governs. \)

\[ t = 2.625" \]

22-4. Find the most economical W14 for a connection with a tensile load of 1200# if there are four lines of bolts (2 in each flange). Each line has four bolts with 1" diameter bolt holes spaced at 3" o.c. and 3" from the end. The lines of bolts are bf/2 " apart.

1. \( P_u = 1200kips \)
2. Gross Yielding: \( A_g > \frac{P_u}{(0.9(F_y))} = \frac{1200k}{(0.9(50ksi))} = 26.67in^2 \)
\[ \text{Must be at least a W14X99 } \ (A = 29.1in^2) \]
3. Choose assumed value for \( U = 0.85 \)
4. Assume \( t_f = .78 \ A_g \geq A_n + Abh = \frac{P_u}{(0.75F_u U)} + (#\text{lines})(\text{bolt hole dia.})(t_f) \)
\[ =1200k/0.75(65ksi)(0.85) + 4(1")(0.78") = 28.96 + 3.12 = 32.08in^2 \]
5. \( r \) cannot be determined without a length.
6. Try W14X120 \( A_g = 35.3, \ t_f = 0.94, \ ry = 3.74, \ bf = 14.7, \ d = 14.5 \)

8. Check the \( U \) value and adjust the equation for \( A_n \) if necessary:
2d/3 = (2/3)(14.5) = 9.67 < 14.7 = bf  ...  U = 0.9
9.  Ag = 1200k/0.75[65ksi](0.9) + 4(1")(tf) = 27.35 + 4tf = 27.35 + 4(.94) = 31.11in² < 35.3 okay
10. Block Shear:
    Agv = (#lines)(shear line length)(tf) = 4(12)(0.94) = 45.12
    Anv = Agv – (#lines)(#bolt holes)(dbh)(tf) = 45.12 – 4(3.5)(1)(0.94) = 31.96
    bf = 14.7,  tension line = (14.7 - 7.35)/2  = 3.675
    Ant = 4(3.675 – .5(1))(0.94) = 11.938in²
    Pu = 0.75[0.6(65ksi)(31.96) + 1.0(65ksi)(11.938)] = 1516.81k > 1200k  okay
    Pu = 0.75[0.6(50)(45.12) + 65(11.938)] = 1597.18k > 1200k  okay
    USE W14X120

22-5.  Repeat problem 22-4 using 7/8" bolt holes spaced 4" o.c. and 3" from the end.

1.  Pu = 1200kips
2.  Gross Yielding:  Ag > Pu/(.9(Fy)) = 1200k/(.9(50ksi)) = 26.67in²
    Must be at least a W14X99  (A = 29.1in²)
3.  Choose assumed value for U = 0.85
4.  Assume tf = .78  Ag ≥ An + Abh = Pu/(0.75FuU) + (#lines)(bolt hole dia.)(tf)
    =1200k/0.75/65ksi/0.85 + 4(.875")(0.78") = 28.96 + 2.73 = 31.69in²
5.  r cannot be determined without a length.
6.  Try W14X109  Ag = 32.0, tf = 0.86,  bf = 14.6,  d = 14.3

8.  Check the U value and adjust the equation for An if necessary:
2d/3 = (2/3)(14.3) = 9.533 < 14.6 = bf  ...  U = 0.9
9.  Ag = 1200k/0.75[65ksi](0.9) + 4(.875")"(tf) = 27.35 + 3.5tf = 27.35 + 3.5(.86) = 30.36in² < 32.0 okay
10. Block Shear:
    Agv = (#lines)(shear line length)(tf) = 4(12)(0.86) = 41.28
    Anv = Agv – (#lines)(#bolt holes)(dbh)(tf) = 41.28 – 4(3.5)(.875)(0.86) = 30.75
    bf = 14.6,  tension line = (14.7 - 7.3)/2  = 3.65
    Ant = 4(3.65 – .5(.875))(0.86) = 11.05in²
    Pu = 0.75[0.6(65ksi)(30.75) + 1.0(65ksi)(11.05)] = 1438.13k > 1200k  okay
    Pu = 0.75[0.6(50)(41.28) + 65(11.05)] = 1467.49k > 1200k  okay
    USE W14X109
23-1. Design a baseplate for a W24X192 column carrying an axial load of \( P_u = 2400 \) k and bearing on a 8' by 8' concrete footing with \( f'_c = 4 \) ksi, \( d = 25.5 \), and \( b_f = 13.0 \).

1. \( P_u = 2400 \) k
2. \( A_2 = 86'(12\text{in}/f)(8')(12\text{in}/f) = 9216\text{in}^2 \)
3. \( A_1 = P_u/[\Phi_c(0.85f'_c)(2)] = 2400/[.6(.85)(4)(2)] = 588.24\text{in}^2 \)
4. \( \text{dbf} = 25.5(13) = 331.5 \). check that \( A_1 > \text{dbf} \)
5. \( 588.24 > 331.5 \) okay
6. Round \( B \) and \( N \) up to whole numbers: \( \sqrt{588.24} = 24.25 \) use 20 X 30:
   \( A_1 = \text{BN} = 600\text{in}^2 \) note: \( \sqrt{(A_2/A_1)} = 3.919 > 2 \) use 30 X 30:
7. Check the bearing strength of the concrete:
   \( P_u < \Phi_cP_p = 0.6(0.85(4)(600))(2) = 2448 > 2400 \text{ k} \) okay
8. Find base plate thickness:
   \( M = [N – 0.95d]/2 = [30 – .95(25.5)]/2 = 2.89 \)
   \( N = [B – 0.80bf]/2 = [20 – .8(13)]/2 = 4.8 \)
   \( n' = [\sqrt{\text{dbf}}]/4 = [\sqrt{25.5(13)}]/4 = 4.55 \)
   \( l = 4.8 \)
9. \( \text{treq} = \sqrt{[2P_u/.9\Phi_yBN]} = 4.8\sqrt{[2(2400)/.9(36)(20)(30)]} = 2.385'' \)
Base Plate: PL 20 X 30 X 2-1/2''

23-2. Design the thickness of a 30'' X 30'' base plate fully covering a pedestal of \( f'_c = 3 \) ksi concrete and supporting a W14X120 columns with an axial load of \( P_u = 1200 \) k, \( d = 14.5 \), and \( b_f = 14.7 \).

Example 23-2: Design the thickness for a base plate of a given size where:
\( P_u = 900 \) k Column W14X90: \( d = 14 \), \( b_f = 14.5 \)
Footing: \( f'_c = 3 \) ksi, 30''X30'' pedestal with baseplate covering pedestal
1. \( P_u = 1200 \) k
2. \( A_2 = 30(30) = 900 \text{ in}^2 \)
3. \( P_p = 0.85f'_cA_1 = .85(3)(900) = 2295 > 1200 \) ... go right to step 7
7. Check the bearing strength of the concrete:
   \( P_u < \Phi_cP_p = 0.6(2295) = 1377 > 1200 \text{ k} \) okay
8. Find base plate thickness
   \( M = [N – 0.95d]/2 = [30 – .95(14.5)]/2 = 8.11 \)
   \( N = [B – 0.80bf]/2 = [20 – .8(14.7)]/2 = 9.12 \)
   \( n' = [\sqrt{\text{dbf}}]/4 = [\sqrt{14.5(14.7)}]/4 = 3.65 \)
   \( l = 9.12 \)
9. \( \text{treq} = \sqrt{[2P_u/.9\Phi_yBN]} = 9.12\sqrt{[2(1200)/.9(36)(20)(30)]} = 2.616'' \)
Base Plate: PL 30 X 30X 2-5/8''

fyi, this plate weighs 669.92# :)
24-1. Find the number of A325 bolts required for a bearing connection with a load $Pu = 300k$ connecting 2 - A36 (Fu = 58ksi) plates, each $\frac{3}{4}''$ thick with 7/8'' bolts spaced at 3'' on center and 3'' from each edge. The plates are 9'' wide and there are 2 rows of bolts. Bolt threads are excluded from shear plane.

1. Determine Bearing strength in one bolt: $Rn = 1.2Lc(t)(Fu)(1 \text{ bolt})$

$Lc = \text{minimum clear distance in direction of force} = \text{smaller of}$

$3 - 1/2'' = 2.5'' \quad \text{or} \quad 3'' - 1'' = 2''$

$Lc = 2''$

$t = 0.75'' = \text{thickness of plate}$,

$d = 0.875'' = \text{diameter of bolt}$

$Rn = 1.2Lc(t)(Fu)(1 \text{ bolt}) = 1.2(2.0)(0.75)(58)(1) = 104.4k/\text{bolt}$

$Rn = 2.4(d)(t)(Fu)(1 \text{ bolt}) = 2.4(0.875)(0.75)(58)(1) = 91.35k/\text{bolt}$

$\Phi Rn = 0.75 \times Rn = 0.75(104.4) = 78.33k/\text{bolt}$

2. Determine Shear Strength in one bolt: $\Phi Rn = \Phi FnAb = 0.75FnAb$

From Table 24.1 For A325-X bolt,

$fnv = 60ksi$

$Ab = \text{area of ONE bolt} = \pi(d/2)^2/4 = (3.14159)(0.875)^2/4 = 0.601in^2$

$\Phi Rn = \Phi FnAb = 0.75(60)(0.601) = 27.06k/\text{bolt}$

3. Determine number of bolts needed:

$Pu = 300k$ (given )

$\Phi Rn = 27.06k/\text{bolt} \times \#\text{bolts} = 300k \quad \#\text{bolts} = 300/27.06 = 11.08 \quad \#\text{round up to 12 bolts}$
24-2. Repeat exercise 24-1 for a slip-critical connection, assuming standard bolt hole size and class A coatings.

1) \( P_u = 300k \)
2) Nominal Strength of one bolt:
   \[ R_n = \mu D_u h_{sc} T_b N_s \]
   \( \mu = 0.35, \ D_u = 1.13, \ h_{sc} = 1.0, \ T_b = 39, \ N_s = 1, \ \Phi = 1 \) (assume serviceability state)
   \[ R_n = 0.35 \times 1.13 \times 1.0 \times 39 \times 1 = 15.42k/bolt \]
   \#bolts required = \( \frac{P_u}{\Phi R_n} = \frac{300k}{1(15.42k/bolt)} = 19.46 \)
   USE 20 Bolts minimum

3) Bearing in bolts
   \[ R_n = 1.5 L_c t u (\#bolts) = 1.5(2)(.75)(58)(20) = 2610k \]
   or
   \[ = 3 d t u (\#bolts) = 3(.875)(.75)(58)(20) = 2283.75k \]
   Check that \( \Phi R_n = 0.75 R_n = 0.75(2283.75) = 1712.81 > P_u = 300 \) ... okay.

4) Shear in bolts
   \[ R_n = F_n A_b (\#bolts) = 60[(3.14159)(.875)^2/4]20 = 721.58 k \]
   Check that \( \Phi R_n = 0.75 R_n = 0.75(721.58k) = 541.19 > P_u = 300k \) ... okay.
   USE 20 bolts: 10 rows of 2.

24-3. Find design load for eccentric connection shown, using ¾" bolts, ¾" thick, A36 plate:

\( e = 3 + 1.5 = 4.5" \)

<table>
<thead>
<tr>
<th>BOLT</th>
<th>h</th>
<th>h²</th>
<th>v</th>
<th>v²</th>
</tr>
</thead>
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<tr>
<td>TL</td>
<td>1.5</td>
<td>2.25</td>
<td>1.5</td>
<td>2.25</td>
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<tr>
<td>TR</td>
<td>1.5</td>
<td>2.25</td>
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<td>BL</td>
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<td>BR</td>
<td>1.5</td>
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</tbody>
</table>

\[ \sum d^2 = \sum h^2 + \sum v^2 = 9 + 9 = 18 \]

Determine the resultant force on each bolt:
\[ H = Mv/\sum d^2 \]
\[ M = 20k(4.5") = 90k-in \]
For all bolts: \( H_1 = 90(1.5)/18 = 7.5k \)

\[ V = Mh/\sum d^2 = 90(1.5)/18 = 7.5k \text{ for all bolts} \]

And vertical force due to Load
\[ = P/\#bolts = 20/4 = 5k \downarrow \]
\[ R = \text{resultant force on bolt} = \sqrt{H^2 + (V + P/\#bolts)^2} \]
BOLT  H  V  R
TL  7.5  -12.5  14.58
TR  7.5  2.5  7.91
BL  7.5  -12.5  14.58
BR  7.5  2.5  7.91  Rmax = 14.58k

Lc = smaller of 3 - 7/8 = 2.125" or 3 -.875/2 = 2.56" ... Lc = 2.125"
Rn = 1.2Lc(t)(Fu)(1 bolt) = 1.2(2.125)(.75)(58) = 110.93k
or
Rn = 2.4(d)(t)(Fu)(1 bolt) = 2.4(.75)(.75)(58) = 78.3k
ΦRn = .75(78.3) = 58.73 > 14.58k ... okay.

24-4.  Find design strength for E70xx weld shown:

1.  Fw = 0.6(70) = 42ksi
2.  Aw = 0.707(0.25)L
3.  Longitudinal welds:  L = 2welds @ 12" each = 24"
   Aw = 0.707(0.25)(24) = 4.24in²
   R_{wl} = 42ksi(4.24in²) = 178.19k
4.  Transverse weld:  L = 6"
   Aw = 0.707(0.25)(6) = 1.06in²
   R_{wt} = 42ksi(1.06in²) = 44.55k
5.  USE LARGER OF
   Rn = R_{wl} + R_{wt} = 178.18 + 44.55 = 222.73k
   or
   Rn = 0.85 R_{wl} + 1.5R_{wt}
   = 0.85(178.19) + 1.5(44.55) = 218.29k
6.  ΦRn = 0.75(218.29k) = 163.71k = LRFD Design Strength
25-1. Determine if a concrete 14" by 30" beam with a simple span of 20' will need reinforcing to carry a 500#/f dead load and a 900#/f live load if f’c = 3,000psi.

\[ Wu = 1.2(500#/f + 14"(30") (150pcf)/144) + 1.6(900#/f) = 2565#/f \]
\[ Mu = wL^2/8 = 2565#/f(20ft)^2/8 = 128,250#-ft = 1,539,000#-in \]
\[ S_x = bh^2/6 = 14(30)^2/6 = 2100in^3. \]
\[ fr = 7.5\sqrt{f’c} = 7.5(1)\sqrt{3,000} = 410.79psi \]
\[ Mcr = frS_x = (410.79psi)(2100in^3) = 826,102.55#-in. \]

or
\[ Z = 2h/3 = 2(30)/3 = 20" \]
\[ A (of compression) = bh/2 = 14(30)/2 = 210in^2. \]
\[ Mcr = ZAfr/2 = 20(210)(410.79)/2 = 826,102.55#-in \]

\[ Mcr < Mu \quad \text{beam will need reinforcing.} \]

25-2. An unreinforced concrete beam has a rectangular cross-section 12" wide by 20" deep. If it is made using concrete with f’c = 4000psi, at what length will it fail under its own weight?

\[ fr = 7.5(1)\sqrt{4000} = 474.34psi \]
\[ Z = 2(20)/3 = 13.33" \]
\[ A = bh/2 = 12(20)/2 = 120in^2. \]
\[ Mcr = ZAfr/2 = 13.33(120)(474.34)/2 = 379,372#-in = Mu = wL^2/8. \]
\[ w = 1.4(12)(20)(150)/144 = 350#/ft \]
\[ 350L^2(12in/ft)/8 = 379,372#-in \]
\[ L = 26.88ft. \]

25-3. Design for flexure: a 14" wide, 24" deep concrete beam with a simple span of 24' and a uniform live load of 960#/f. f’c =4000psi and fy = 60,000psi

1. f’c=4,000psi, fy=60,000psi, b=14" and h=24".
2. \[ Wu = 1.2(14)(24)(150)/144 + 1.6(960) = 2323.5#/f \]
   \[ Mu = 2323.5#/f(24ft)^2/8 = 167,292#-ft = 2,007,504#-in \]
3. Estimate \( d = h – 3 = 24-3 = 21" \)
4. Assume \( \Phi = 0.9 \)
5. \[ As = \frac{[0.85f’c bd/ fy][1 – \sqrt{1 – 2Mu/\Phi f’c bd^2}]}{[0.85(4000)(14)(21)]/60000} = \frac{[0.85(4000)(14)(21)]/60000}[1 – \sqrt{1 – 2(2,007,504)/(0.9(0.85)(4000)(14)(21)^2)\] = 1.876in^2 \]
6. \[ As \min = bd(3\sqrt{f’c}/fy) = 14(21)(3)(\sqrt{4000})/60000 = 0.93 < 200bd/fy = 0.98 \]
   \[ As \min = 0.98in^2. \]
7. Use 2 - #9, \( As = 2.0, \ bmin = 7.0 \)
90
d = 24-1.5-0.375 -1.128/2 = 21.56"
8. \(a = \frac{f_y A_s}{(0.85 f'_c b)} = 60000(2)/0.85/4000/14 = 2.52"\) \(c = a/\beta_1 = 2.52/0.85 = 2.97"\)
9. Check \(\epsilon_t = \frac{0.003(d - c)}{c} = \frac{0.003(21.56-2.97)}{2.97} = 0.019 > 0.004...\) okay
10. Check \(\Phi = 0.9\)
11. \(\Phi[f_y A_s(d - f_y A_s/(1.7 f'_c b))] = 0.9(60000)(2)(21.56-60000(2)/1.7/4000/14) = 2,192,345#-in > \mu = 2,007,504#-in\)

USE 2- #9

25-4. Design for flexure: a 16" wide by 30" deep concrete beam with a simple span of 32' to carry 2 point loads, each 3000# dead load, evenly spaced. \(f'_c = 4000\text{psi}\) and \(f_y = 60,000\text{psi}\)

There is no live load: governing eqtn = \(U = 1.4D\)
1. \(f'_c = 4,000\text{psi}, f_y = 60,000\text{psi}, b = 16"\) and \(h = 30"\).
2. \(W_u = 1.4(16)(30)(150)/144 = 700#/f\)
   \(P_u = 1.4(3000) = 4200#\)
   \(\mu = 700#/f(32ft)^2/8 + 4200(32)/3 = 134,400#-ft = 1,612,800#-in\)
3. Estimate \(d = h - 3 = 30 - 3 = 27"\)
4. Assume \(\Phi = 0.9\)
5. \(A_s = \frac{0.85 f'_c b d}{f_y} [1 - \sqrt{1 - 2\mu/\Phi 0.85 f'_c b d^2}] = \frac{0.85(4000)(16)(27)}{60,000}[1 - \sqrt{1 - 2(1,612,800)/0.9(0.85)(4000)(16)(27)^2}] = 1.132\text{in}^2\)
6. \(A_s \text{min} = b d (3\sqrt{f'_c c})/f_y = 16(27)(3)(\sqrt{4000})/60000 = 1.366 < 200bd/fy = 1.44...\) \(A_s \text{min} = 1.44\text{in}^2\).
   \(A_s \text{min} > A_s\).
7. Use 2 - #8, \(A_s = 1.571,\) \(b \text{min} = 7.0\)
   \(d = 30-1.5-0.375 - 1/2 = 27.625"\)
8. \(a = \frac{f_y A_s}{(0.85 f'_c b)} = 60000(2)/0.85/4000/16 = 2.21"\) \(c = a/\beta_1 = 2.21/0.85 = 2.60"\)
9. Check \(\epsilon_t = \frac{0.003(d - c)}{c} = \frac{0.003(27.625-2.6)}{2.6} = 0.029 > 0.004...\) okay
10. Check \(\Phi = 0.9\)
11. \(\Phi[f_y A_s(d - f_y A_s/(1.7 f'_c b))] = 0.9(60000)(2)(27.625-60000(2)/1.7/4000/16) = 2,864,382#-in > \mu = 1,612,800#-in\)

USE 2- #8
25-5. Design for flexure: a 12" wide beam with a simple span of 26' to carry PD = 1000# and PL = 2000# at the center of the span. f'c = 4,000psi and fy = 60,000psi.

1. \( f'c = 4,000\text{psi}, \) fy = 60,000psi, b = 10"
2. \( Pu = 1.2(1000) + 1.6(2000) = 4400\)#
   \( Mu = 4400(26)/4 = 28,600\text{#-f} = 343,200\text{#-in}.\)
3. Assume \( \Phi = 0.9 \)
4. 
   \[ d = \sqrt{\frac{Mu}{0.153\Phi f'c b}} = \sqrt{\frac{343,200}{0.153(0.9)(4000)(12)}} = 7.2" \]
5. 
   \[ d/b = 7.2"/120" = 0.06" \text{ and } 1.5 < d/b < 2.2 \text{ ... is not satisfied.} \]
   reduce the value of b to b=6.5"

4A. 
   \[ d = \sqrt{\frac{Mu}{0.153\Phi f'c b}} = \sqrt{\frac{343,200}{0.153(0.9)(4000)(6.5)}} = 9.79" \]
5A. 
   \[ d/b = 9.79/6.5 = 1.506 \text{ > 1.5 ... okay} \]
6. Estimate \( h = d + 2.5 = 9.79" + 2.5" = 12.29" \). Round up to 12.5".
7. \( W_{bm} = 150\text{pcf}(6.5"/12)(12.5"/12) = 84.64#/f \)
   \( Wu = 1.2(84.64#/f) = 101.56#/f \)
8. \( Mu = wL^2/8 + PL/4 = 101.56#/f (26')^2/8 + 4400(26)/4 = 37,181.82#-f = 446,181.84#-in \)
9. \( R = 0.85f'cbd = 0.85(4000)(6.5)(9.79) = 216,359 \)
   \( As = \left[R/fy\right][1 - \sqrt{1 - 2Mu/\Phi R d}] = [216,359/60,000][1 - \sqrt{2(446,181.84)/(0.9(216,359)(10.19))}] = 1.19\text{in}^2 \)
10. Check that \( As \geq As_{min} = bd(3\sqrt{f'c})/fy = 6.5(9.79)(3\sqrt{4000})/60,000 = 0.201 \geq 200bd/fy = 200(6.5)(9.79)/60,000 = 0.212 \)
    As min = 0.212 As > As min ... okay
11. Use 2 - #7: \( As = 1.203, b_{eq} = 6.5 = b. \)
12. \( c = a/\beta_1 = 3.27/0.85 = 3.84" \)
13. Calculate \( d_{actual} = 12.5 - 1.5 - 0.375 - 1.128/2 = 10.061" \)
14. Check \( \epsilon_t = 0.003(d - c)/c = 0.003(10.06 - 3.84)/3.84 = 0.0048 > 0.004 \text{ ... okay} \)
15. \( \Phi = 0.65 + (\epsilon_t - 0.002)(250/3) = 0.883 \)
16. \( \Phi f_y As(d - a/2) = 0.883\times[60000(1.203)(10.06 - 3.27/2)] = 1,206,182\text{-in allowable moment} > 446\text{-in actual moment ... beam is okay} \)
USE 6.5" X 12.5" beam with 2-#7.
Design the lightest beam (ignore weight of reinforcement steel) with a maximum width of 16" to carry 1500#/f uniform dead load and a 2000#/f uniform live load over a span of 16'.

1. $f'_c = 4000$psi, $f_y = 60000$psi, try $b = 11''$
2. $W_u = 1.2(1500) + 1.6(2000) = 5000#/f$
   $M_u = 5000#/f(16')^2/8 = 160,000#\cdot ft = 1,920,000#\cdot in.$
3. Assume $\Phi = 0.9$
4. $d = \sqrt{M_u / {[0.153f'_c b]}} = \sqrt{1,920,000 / {[0.153(0.9)(4000)(11)]}} = 17.8''$
5. $d/b = 17.8''/11'' = 1.62$ and $1.5 < 1.62 < 2.2 \quad \Rightarrow \quad b$ is okay.
6. Estimate $h = d + 2.5 = 17.8'' + 2.5'' = 20.3''$. Round up to 21''.
7. $W_{bm} = 150pcf(11''/12)(21''/12) = 240.63#/f$
   $W_u = 1.2(240.63#/f) + 5000 = 5288.75#/f$
8. $M_u = W L^2/8 = 5288.75#/f (16')^2/8 = 169,240#\cdot f = 2,030,880#\cdot in$
9. $R = 0.85f'_c b d = 0.85(4000)(11)(17.8) = 665,720$
   $A_s = \frac{R}{f_y} = \frac{665,720}{60,000} = 11.12$
   $b_{req} = 10 < 11 = b$.
10. Use $3$ -#11 $A_s = 4.684$, $b_{req} = 10 < 11 = b$.
11. $a = f_y A_s / [0.85f'_c b] = 60000(4.684) / [0.85(4000)(11)] = 7.51''$
    $c = a/\beta_1 = 7.51/0.85 = 8.85''$
12. Calculate $d_{actual} = 21 - 1.5 - 0.375 - 1.41/2 = 18.42''$
13. $E_t = .003(d - c)/c = .003(18.42 - 8.85)/8.85 = .00324 < 0.004 \quad \Rightarrow \quad NO \ GOOD$
    must increase $d$.
    Increase $h = 24''$, $d = 21.42''$
14. $E_t = .003(d - c)/c = .003(21.44 - 8.85)/8.85 = 0.00426 > 0.004 \quad \Rightarrow \quad OKAY$
15. $\Phi = 0.65 + (.00426 - 0.002)(250/3) = 0.839$
16. $\Phi f_y A_s (d - a/2) = .839[60000(4.684)(21.42 - 7.51/2)] = 4,165,275.57#\cdot in$ allowable moment > $2,030,80#\cdot in$ actual moment \quad \Rightarrow \quad beam \ is \ okay
USE 11" X 24" beam with 3-#11.
26-1: Find the allowable service live load in psf for an 8" deep, one way slab with a 12ft span, 3/4" cover, with f'c = 4000psi and fy = 60,000psi and longitudinal steel = #5 @ 9" o.c.

1. \( As = (0.307in^2)(12''/f)/9'' = 0.409in^2/f \)
   \( d = 8 - .75 - .625/2 = 6.94'', b = 12'' \)
2. \( a = fyAs/(0.85f'cbd) = 60,000psi(0.409in^2/f)/[0.85(4000psi)(12''/f)] = 0.601'' \)
3. \( Mn = fyAs(d - a/2) = 60,000psi(0.409in^2/f)(6.94 - 0.601/2) = 162,933.33#-in \)
4. Check \( As \geq As\ min = 0.0018bh = 0.0018(12''/f)(10'') = 0.216in^2/f < 0.409in^2/f \) ... okay
5. \( c = a/\beta1 = 0.601/0.85 = .707 \)
   \( \epsilon t = 0.003(d - c)/c = .003(6.94 - 0.707)/.707 = 0.026'' \)
6. \( 0.026 \geq 0.004 \)
7. \( \Phi = 0.90 \) because \( 0.026 > 0.005 \)
8. \( Mu = \Phi Mn = 0.9(162,933.33#-in) = 146,640#-in = 12,220#-f \)
   \( Mu = 12,220#-f = w(12'')^2/8 \) ... \( wu = 12,200(8)/12^2 = 677.78#/f \)
One foot section of slab weight = \( wbm = 150pcf(8''/12''/f)(1'') = 100#/f \)
   \( wu = 1.2(100#/f) + 1.6(111) = 677.78#/f \) ... \( LL = 348.61#/f \) per foot of slab

26-2: Design a slab to span 14ft and carry a Live Load = 120psf where deflection is not checked. f'c = 3,000psi  fy = 40,000psi use # 5 rebars (A = .307)

1. \( h_{min} = L/20(4 + 40/100) = 14(12)(.8)/20 = 6.72'' \) round up to 7"
2. \( wu = 1.2(150)(7/12)(12/12) + 1.6(120) = 297#/f \)
   \( Mu = 297(14)^2/8 = 7276.5#-f = 87,318#-in \)
3. \( d = 7– 1.12 = 5.88'' \)
4. Assume \( \Phi = 0.9 \)
5. \( As = (0.85f'cbd/fy)[1 – \sqrt{1 - 2 Mu/ \Phi(0.85f'cbd^2)}] \)
   \( = 0.85(3000)(12)(5.88)/40000[1 – \sqrt{1 - 2(87,318)]/[0.9(0.85)(3000)(12)(5.88)^2}] = 0.433 \)
6. \( a = fyAs/(0.85f'cb) = 40,000(0.433)/(0.85(3000)(12)) = 0.566'' \)
   \( c = .566/.85 = 0.666 \)
   \( \epsilon t = 0.003(d – c)/c = .003(5.88 - 0.666)/.666 = .0234 > .005 \) ... \( \Phi = 0.9 \)
7. \( As\ min = .002bh = .002(12)(7) = 0.168 \)
8. Longitudinal steel spacing: \( s = (.307)(12/.433) = 8.51'' \) round down to 8.5"
10. check maximum spacing of 5h or 18".
   \( 5(7) = 35'' > 18'' \) ... max. spacing = 18"
Answer:
Temperature steel: #5 @ 18"
Longitudinal steel: #5 @ 8.5"
Slab thickness = 7"
26-3: Design a slab to span 15ft and carry a Live Load = 90psf where deflection is not checked. $f'c = 4,000psi$ $fy = 60,000psi$ use # 5 rebars

1. $h_{min} = L/20(4 + 40/100) = 15(12)(.8)/20 = 7.2''$ round up to 7.5''
2. $w_u = 1.2(150)(7.5/12)(12/12) + 1.6(90) = 256.5#/$f
   
   $Mu = 256.5(15)^2/8 = 7,214.06#-f = 86,568.75#-in$
3. $d = 7.5 - 1.12 = 6.38''$
4. Assume $\Phi = 0.9$
5. $As = [0.85f'cbd/fy][1 - \sqrt{1 - 2Mu/\Phi(0.85f'cbd)^2}]$
   
   $c = [0.85(4000)(12)(6.38)/6000][1 - \sqrt{1 - 2(86,568.75)/0.9(0.85)(4000)(12)(6.38)^2}] = 0.259$
6. $a = fyAs/(0.85f'cb) = 60,000(0.259)/(0.85)(4000)(12) = 0.381''$
   
   $c = 0.381/0.85 = 0.448$
7. $\epsilon_t = 0.003(d - c)/c = 0.003(6.38 - 0.448)/0.448 = 0.0397 > .005 \quad \Phi = 0.9$
8. As min = .0018bh = .0018(12)(7.5) = 0.162
9. Longitudinal steel spacing: $s = (.307)(12/.259) = 14.22''$ round down to 14''
10. check maximum spacing of 5h or 18''

5(7.5) = 37.5'' > 18'' ... max. spacing = 18''

Answer:
Temperature steel: #5 @ 18''
Longitudinal steel: #5 @ 14''
Slab thickness = 7.5''

26-4: Design a slab with minimum thickness to span 14ft and carry a LL = 120psf where deflection will be checked. $f'c = 3,000psi$ $fy = 40,000psi$ use # 5 rebars

1. assume $h = 6''$ for weight
2. $w_u = 1.2(150)(6/12)(12/12) + 1.6(120) = 282#/$f
   
   $Mu = 282(14^2)/8 = 6909#-f = 82,908#-in$
3. Assume $\Phi = 0.9$
4. $d = \sqrt{Mu/.153\Phi f'cb} = \sqrt{82,908/(0.153(0.9)(3000)(12))] = 4.09$
5. Estimate $h = 4.09 + 1.12 = 5.21$ and round up to 5.25'', $d = 5.25 - .75 - .5(0.625) = 4.19''$
6. $w_u = 1.2(150)(5.25/12)(12/12) + 1.6(120) = 270.75#/$f
   
   $Mu = 270.75(14^2)/8 = 6633.38#-f = 79,600.5#-in$
7. $As = [0.85f'cbd/fy][1 - \sqrt{1 - 2Mu/\Phi(0.85f'cbd)^2}]$
   
   $c = [0.85(3000)(12)(4.19)/40,000][1 - \sqrt{1 - 2(79,600.5)/0.9/85/3000/12/4.19^2}] = 0.58$
8. $a = fyAs/(0.85f'cb) = 40000(0.58)/85/3000/12 = 7.58 \quad c = 7.58/0.85 = 0.89$
   
   $\epsilon_t = 0.003(d - c)/c = 0.003(4.19 - 0.89)/0.89 = 0.0111 > .005 \quad \Phi = 0.9$
9. As min = .002bh = .002(12)(5.25) = 0.126
10. Longitudinal steel spacing: $s = (bar area)/(12/As) = .307(12)/.58 = 6.35''$ round down to 6''
11. Temperature steel: $s = (bar area)/(12/As min) = .307(12)/.126 = 29.23$
12. $5h = 5(5.25) = 26.25$ or 18''.

Answer:
Temperature steel: #5 @ 18''
Longitudinal steel: #5 @ 6''
Slab thickness = 5.25''
26-5: Design a slab with minimum thickness to span 15’ and carry a LL = 90psf where deflection will be checked. \( f’c = 4,000 \text{psi} \) \( fy = 60,000 \text{psi} \) use #5 rebars

1. Assume \( h = 6” \) for weight
2. \( w_u = 1.2(150)(6/12)(12/12) + 1.6(90) = 234#/f \)
   \[ Mu = 234(15^2)/8 = 6581.25#-f = 78,975#-in \]
3. Assume \( \Phi = 0.9 \)
4. \( d = \sqrt{[Mu/.153\Phi f’c b]} = \sqrt{[78,975/(.153)(.9)(4000)(12)]} = 3.46” \)
5. Assume \( h = 3.46 + 1.12 = 4.58 \) and round up to 4.75” \( d = 4.75 - .75 - .5(625) = 3.69” \)
6. \( w_u = 1.2(150)(4.75/12)(12/12) + 1.6(90) = 215.25#/f \)
   \[ Mu = 215.25(15^2)/8 = 6053.91#-f = 72,646.88#-in \]
7. \( As = [.85f’cbd/fy][1 - \sqrt{[1 - 2Mu/\Phi .85f’cbd^2]}] = [.85(4000)(12)(3.69)/60,000][1 - \sqrt{[1 - 2(72,646.88)/.9/.85/4000/12/3.69^2]} = 0.462 \]
8. \( \alpha = fyAs/(.85f’cb) = 60000(.462)/.85/4000/12 = .679 \)
   \[ \varepsilon_t = .003(d-c)/c = .003(3.69-.799)/.799 = .0109 > .005 \]... \( \Phi = 0.9; \)
9. \( As_{min} = .0018bh = .0018(12)(4.75) = 0.103 \)
10. Longitudinal steel spacing: \( s = (bar \text{ area})(12/As) = .307(12)/.462 = 7.95” \) round down to 7.5”
11. Temperature steel: \( s = (bar \text{ area})(12/As \text{ min}) = .307(12)/.103 = 35.77 \)
12. \( 5h = 5(4.75) = 23.75 \) or 18”.

Answer: Temperature steel: #5 @ 18”
Longitudinal steel: #5 @ 7.5”
Slab thickness = 4.75”
26-6: Design a continuous slab for the plan shown below if the floor carries a LL of 90psf and a dead load of 15psf. \( f'c = 4,000 \text{ psi} \) \( fy = 60,000 \text{ psi} \) use #5 rebars

1. Determine minimum slab thickness:
   \( 20'(12)/24 = 10" \), \( d = 10 – 1.12 = 8.88" \)
2. \( w_u = 1.2(15 + 150\text{pcf}(10"/12)(12"/12)) + 1.6(90\text{psf})(1') = 312\#/f \)
3. \( wL^2(12"/f) = 312(20')(12) = 1,497,600\#-\text{in} \)
4. Calculate \( Mu \) for each location: see Table
5. Calculate \( As = \left(0.85f'cbd/fy\right)[1 – \sqrt{1 – 2Mu/0.85f'cbd^2}] \)
6. Calculate \( a, c, \epsilon_t \) each case.
7. \( As \text{ min} = .0018bh = .0018(12)(10) = .216 \)
8. Longitudinal steel spacing: \( s = (0.307)(12/As) \)
9. Temperature steel: \( s = (0.307)(12/0.216) = 17.06 \) round down to 17"
10. check maximum spacing of 5h or 18".
    \( 5h = 5(10) = 18" > 18" \) max. spacing = 17"
11. Check \( \Phi V_n = \Phi 2\sqrt{f'cbd} > V_u = 1.15wL/2 \). If not, increase \( h \) and go back to step 1. \( \Phi V_n = (.75)2\sqrt{4000(12)(8.88)} = 10,109.17\# > 1.15(312)(20)/2 = 3588\# \) ... okay

<table>
<thead>
<tr>
<th>Location</th>
<th>( Mu = wL^2/\text{ft} )</th>
<th>( c )</th>
<th>( As \text{ min} )</th>
<th>( As \text{ used} )</th>
<th>( S_{\text{long}} )</th>
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<tr>
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<td>0.4718</td>
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<tr>
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<td>L</td>
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<td>( Yu )</td>
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26-7: Design a continuous slab for the plan shown below if the floor carries a LL of 90 psf and a dead load of 15 psf. \( f'c = 4,000 \text{psi} \) \( \text{fy} = 60,000 \text{psi} \) use # 5 rebars

1. Determine minimum slab thickness:
   \( 10'(12)/24 = 5", \ d = 5 - 1.12 = 3.88" \)

2. \( w_u = 1.2(15 + 150\text{pcf}(5"/12)(12"/12)) + 1.6(90\text{psf})(1') = 237#/f, \)

3. \( wL^2(12"/f) = 237(10^2)(12) = 284,400\#\text{-in} \)

4. Calculate \( Mu \) for each location: see Table

5. Calculate \( As = \lfloor .85f'cbd/fy \rfloor [1 - \sqrt{1 - 2Mu/\Phi.85f'cbd^2}] = 2.638[1 - \sqrt{1 - (\text{Mu}/276,398.78)}] \)

6. Calculate \( a, c, \epsilon_t \) each case.

7. \( As \text{ min} = .0018bh = .0018(12)(5) = .108 \)

8. Longitudinal steel spacing: \( s = (.307)(12/As) \)

9. Temperature steel: \( s = (.307)(12/.108) = 34.12 \text{ round down to 34"} \)

10. check maximum spacing of 5h or 18".
    \( 5h = 5(10) = 18" > 18" \ldots \text{max. spacing} = 18" \)

<table>
<thead>
<tr>
<th>Location</th>
<th>( \text{Mu} )</th>
<th>( \text{As} )</th>
<th>( a )</th>
<th>( c )</th>
<th>( \epsilon_t )</th>
<th>( \text{As min} )</th>
<th>( \text{As required} )</th>
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<tr>
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<td>4417.069</td>
<td>4417.069</td>
<td>4417.069</td>
<td>4417.069</td>
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\[ wL^2(12"/f) = 237(10^2)(12) = 284,400\#\text{-in} \]
Find $\Phi Mn$ for the beam shown below. $f'c = 4$ksi, $fy = 60$ksi

$$\text{As} = 5 \text{in}^2 \quad \text{As'} = \text{As}_2 = 2.0 \text{in}^2$$

$$\text{As}_1 = \text{As} - \text{As}_2 = 5 - 2 = 3 \text{in}^2$$

Assume condition 1: $f_s = f_s' = fy$

$$a = \text{As}_1 \quad \frac{fy}{\beta f'c b} = \frac{3(60)}{[.85(4)(16)]} = 3.309'' \quad c = \frac{3.309}{.85} = 3.893''$$

$$d = 30''$$

Using similar triangles:

$$\varepsilon_s' = \frac{.003(3.893 - 2.5)}{3.893} = .00107 < .00207$$

$$\varepsilon_t = \frac{.003(30 - 3.893)}{3.893} = 0.020 > .00207$$

... Condition 2

$$NT = NC1 + NC2 \quad \text{Asfy} = .85 f'cba + f_s' As'$$

$a$ and $f_s'$ have changed because the assumption that the beam was in Condition 1 was wrong.

$$a = \beta c \quad f_s' = \frac{\varepsilon'}{c E_s} = [.003(c - d')/c] E_s$$

The only unknown is $c$. Solve for $c$ by substituting these equations into the original and forming a quadratic equation

$$\text{Asfy} = (.85 f'c)(b)(a) + f_s' As'$$

$$\text{Asfy} = .85 f'c b \beta c + .003 E_s As' - E_s d'/c$$

As $5$, $fy = 60$ksi, $f'c = 4$ksi, $b = 16''$, $\beta = 0.85$, $d' = 2.5''$, $E_s = 29,000$ksi, $As' = 2.0$

Note: Be careful to use consistent units: If using $E_s = 29,000$ksi, use $f'c$ and $fy$ in ksi.

$$\text{Asfy} = 5(60) = (.85)(4)(16)(.85)(c) + [.003(c - 2.5)/c](29,000)(2)$$

$$300 = 46.24c + 174 (c - 2.5)/c = 46.24c + 174 - 437.5/c$$

$$46.24c^2 - 126c - 437.5 = 0 = c^2 - 2.725c - 9.462$$

Use quadratic equation formula:

$$c = 2.725/2 \pm \frac{.5 \sqrt{(2.725^2 + 4(9.462))}}{1.363 \pm 3.364} = 4.727''$$

Check that assumptions are correct.

$$f_s' = [.003(c - d')/c] E_s$$

$$= [.003(4.727 - 2.5)/4.727][29,000]$$

$$= 40.988 < fy = 60$ksi ... assumption is correct

Knowing $c = 4.727''$, check $\varepsilon_t \geq 0.004$

$$\varepsilon_t = .003(dt - c)/c = .003(30 - 4.727)/4.727 = 0.0160 > 0.004 \quad ... \quad \text{okay}$$

$$\Phi = 0.9 \text{ because } .0160 > .005$$

Solve for $\Phi Mn$:

$$d = 30'' \quad c = 4.727. \quad a = .85(4.727) = 4.018$$

$$Mn_1 = NC_1 Z_1 = NC_1 (d - a/2) = .85 f'cab(d - a/2)$$

$$= (.85)(4ksi)(.85)(4.018)(16)(30 - 4.018/2) = 5200.51 \text{k-in} = 433.38k-f$$

$$Mn_2 = NC_2 Z_2 = NC2(d - d') = As'f_s'(d - d')$$

$$= 2(40.988)(30 - 2.5) = 2254.34 \text{k-in} = 187.86k-f$$

$$\Phi Mn = 0.9(433.38 + 187.86) = 559.12k-f$$

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27-2: Find $\Phi M_n$ for the beam shown below. $f'c = 5 $ksi, $fy = 60 $ksi

Assume all steel yields.
$A_{s2} = A_{s'} = 1.571 \text{in}^2$
$A_s = 6.283 \text{in}^2, A_{s1} = 6.283 - 1.571 = 4.712 \text{in}^2$
$a = 4.712(60,000)/(0.85(5000)(14)) = 4.752''$
$c = a/ \beta$
for $f'c > 4000psi$. $\beta = 0.85 - 0.05(f'c - 4000)/1000 \geq 0.65$
$c = 4.752/0.8 = 5.94''$
$\varepsilon_s' = 0.003(c - d')/c = 0.003(5.94 - 2.5)/5.94 = 0.00174 < 0.00207$
The compression steel has not yielded ($\varepsilon_s' < \varepsilon_y$)
$\varepsilon_t = 0.003(dt - c)/c = 0.003(26 - 5.94)/5.94 = 0.0101 > 0.00207$
The tensile steel had yielded ($\varepsilon_t > \varepsilon_y$)
Condition 2 exists:
$N_T = N_{C1} + N_{C2} \ldots$

Asfy = $0.85f'c b \beta + [.003(c - d')/c]E_s A_{s'}$ where:
As = 6.283, fy = 60ksi, $f'c = 5 $ksi, b = 14'', $\beta = 0.8$, d' = 2.5'', Es = 29,000ksi, $A_{s'} = 1.571$
Note: Be careful to use consistent units: If using Es = 29,000ksi, use $f'c$ and fy in ksi.
Asfy = 6.283(60) = ($0.85(5)(14)(0.8(c) + [.003(c - 2.5)/c](29,000)(2)$

$376.98 = 47.6c + 174 - 435/c$

Use quadratic equation formula:
$c = 4.264/2 + .5\sqrt{(4.264^2 + 4(9.139))} = 2.132 + 3.699 = 5.83''$

Check that assumptions are correct.
$\varepsilon_s' = [.003(c - d')/c]E_s$

$c = 5.83'', check \varepsilon_t \geq 0.004$

Knowing c = 5.83'', check $\varepsilon_t = 0.003(dt - c)/c = 0.003(26 - 5.83)/5.83 = 0.0104 > 0.004 \ldots$ okay
$\Phi = 0.9$ because .0104 > .005

Solve for $\Phi M_n$:
d = 26'' - 2.5''/2 = 24.75''

$M_{n1} = NC_s Z_s = NC_s (d - a/2)$

$= 0.85f'c b (d - a/2)$
$= 0.85(5)(5)(14)(24.75 - 0.8(5.83)/2)$

$= 6221.17\text{k-in} = 518.43\text{k-ft}$

$M_{n2} = NC_s Z_s = NC_s (d - d') = A_s \varepsilon_s' (d - d')$

$= 2(49.69)(24.75 - 2.5) = 2211.21\text{k-in} = 184.27\text{k-ft}$

$\Phi M_n = 0.9(367.06 + 184.27) = 496.19\text{k-ft}$
27-3:  Design the steel for a beam with:  \( \mu = 450 \text{k-f}, \ h = 26", \ \text{f'c} = 3 \text{ksi}, \ \text{fy} = 60 \text{ksi}, \ \text{d}' = 2.5" \\

1.  \( \mu = 450 \text{k-f} = 5400 \text{k-in} \)
2.  Assume \( d = h - 3 = 26 - 3 = 23" \)
3.  Assume \( \Phi = 0.9 \)
4.  \( A_s = \left[0.85 f'c d b / f_y\right] \left[1 - \sqrt{1 - 2 \mu / \Phi (0.85 f'c b d)^2}\right] \)
   \[ \left[0.85(3)(14)(23)/60\right] \left[1 - \sqrt{1 - 2(5400)/(0.9)(0.85)(3)(14)(23)^2}\right] = 5.42 \text{in}^2 \]
5.  \( c = a/\beta_1 = 9.11/0.85 = 10.72" \)
   \[ \epsilon_t = 0.003(d - c)/c = 0.003(23 - 10.72)/10.72 = 0.0034 < 0.004 \text{ ... beam needs to be enlarged or}
   \text{needs double reinforcement.} \]
6.  \( \Phi M_n_1 = 0.9 [0.85 f'c a b (d - a/2)] = 0.9(0.85)(3)(7.33)(14)(23 - 7.33/2) = 4553.64 \text{k-in} \)
7.  \( A_{s_1} = 0.85 f'c a b / f_y = 85(3)(7.33)(14)/60 = 4.36 \text{in}^2 \)
8.  \( A_{s_2} = \Phi M_n_2 / \Phi (d - d') = 4553.64 \text{k-in}/0.9(23 - 2.5) = 45.87 \text{k} \)
9.  \( \alpha = 7.33" \text{from step 6)} \... \ c = a/\beta_1 = 7.33/0.85 = 8.62" \)
   \[ \epsilon_s' = 0.003(8.62 - 2.5')/8.62 = 0.00212 > \epsilon_y = 0.00207 \text{ ... fs' = fy = 60ksi.} \]
10.  \( A_s = NC_2 / fs' = 45.87k/60ksi = 0.765 \text{in}^2 \text{ use 2-#6 for As' = 0.884} \)
11.  \( A_s = A_{s_1} + A_{s_2} = 4.36 + 0.884 = 5.244 \text{ use 4 - #11 for As = 6.246} \)
12.  Check actual \( d > 23 \) (assumed \( d \))
   \( d = 26 - 1.5 - .375 - 1.41/2 = 23.42 > 23 \text{ ... okay} \)
   \( A_s' = 0.884, \ As = 6.246 \)
13.  Check \( \epsilon_s', \epsilon_t \text{ and } \Phi \text{ using selected steel:} \)
   \( \alpha = (A_s - A_{s_1})f_y / 0.85f'c b = (6.246 - 0.884)(60)/0.85/3/14 = 9.01 \text{ ...} \)
   \( c = 9.01/0.85 = 10.60" \)
   \[ \epsilon_s' = 0.003(10.60 - 2.5)/10.60 = 0.00229 > 0.00207 \text{ ... fs' = fy = 60ksi} \]
   \( \epsilon_t = 0.003(23.42 - 10.6)/10.6 = 0.0036 < 0.004 \text{ ... NO GOOD} \)
   Add more steel on top  use 3-#6  \( As' = 1.325, \ As = 4.36 + 1.325 = 5.685 \)
   \( \alpha = (6.246 - 1.325)(60)/0.85/3/14 = 8.27, \ c = 9.73 \)
   \( \epsilon_t = 0.003(23.42 - 9.73)/9.73 = 0.00422 > 0.004 \text{ ... okay} \)
   \( \Phi = 0.65 + (.00422 - 0.002)(250/3) = 0.835 \)
14.  Check \( \Phi M_n > \mu: \)
   \( M_n_1 = A_s f_y (d - a/2) = (4.36)(60)(23.42 - 8.27/2) = 5044.96 \text{k-in} \)
   \( M_n_2 = A_{s_1} f_s' (d - a') = 1.325(60)(23.42 - 2.5) = 1663.14 \text{k-in} \)
   \( \Phi M_n = 0.835(5044.96 + 1663.14) = 5601.26 > \mu = 5400 \text{k-in okay.} \)
Answer:  Use 4 - #11 on the bottom, 3-#6 on the top.
27-4: Design the steel for a beam with: \( \mu = 600\text{k-f}, b = 16'', h = 30'', f'c = 4\text{ksi}, f_y = 60\text{ksi}, d' = 2.5'' \)

1. \( \mu = 600\text{k-f} = 7200\text{k-in} \)
2. Assume \( d = h - 3 \) ... \( d = 30 - 3 = 27'' \)
3. Assume \( \Phi = 0.9 \)
4. \( A_s = \left[0.85 f'c b d / f_y \right] \left[1 - \sqrt{1 - 2\mu / \Phi \cdot 0.85 f'c b d^2} \right] = \left[0.85 \cdot 4 \cdot 16 \cdot 27 / 60 \right] \left[1 - \sqrt{1 - 2 \cdot 7200 / (0.9 \cdot 0.85 \cdot 4 \cdot 16 \cdot 27)^2} \right] = 5.57\text{in}^2 \)
5. \( a = f_y A_s / (0.85 f'c b) = 60 \cdot (5.57) / (0.85 \cdot 4 \cdot 16) = 6.14'' \)
\( c = a / \beta_1 = 6.14 / 0.85 = 7.23'' \)
\( \varepsilon_t = 0.003 (d - c) / c = 0.003 (27 - 7.23) / 7.23 = 0.0082 > 0.004 \) ... no double reinforcement.

6. \( A_{\text{min}} = b d (3 \sqrt{f'c}) / f_y = 16 \cdot 27 \cdot (3 \sqrt{4000}) / 60000 = 1.366 < 200 b d / f_y = 1.44 \) ... \( A_{\text{min}} = 1.44\text{in}^2. \)
\( A_{\text{min}} > A_s. \)
7. Use \( 6 - \#9, \quad A_s = 6, \quad b_{\text{min}} = 15.5 \)
\( d = 30 - 1.5 - 0.375 - 1.128 / 2 = 27.56'' \)
8. \( a = f_y A_s / (0.85 f'c b) = 60000 \cdot 6 / 0.85 / 4000 / 16 = 6.62'' \)
\( c = a / \beta_1 = 6.62 / 0.85 = 7.79'' \)
9. Check \( \varepsilon_t = 0.003 (d - c) / c = 0.003 (27.56 - 7.79) / 7.79 = 0.0076 > 0.004 \) ... okay
10. Check \( \Phi = 0.9 \)
11. \( \Phi [f_y A_s (d - f_y A_s / (1.7 f'c b))] = 0.9 [60\text{ksi}] (6) (27.56 - 60) / 1.7 / 4\text{ks}/16] = 7857.38\text{k-in} > \mu = 7200\text{k-in} \)

USE \( 6 - \#9 \)
27-5: Find the $\Phi$ Mn for the T-beam shown below. $f'c = 3\text{ksi}$ and $f_y = 60\text{ksi}$. The span is 20ft and the center-to-center beam spacing is 5'.

1. Find Effective Flange Length:
   $L/4 = 20(12)/4 = 60''$
   bw + 16 hf = 16 + 80 = 96''
   beam spacing = 60''

   use $b = 60''$

2. As min = .0033bwd = .0033(16)(21) = 1.11 < 4.684 (3-#11)...okay

3. Assume the steel yields. Find NT:
   $NT = Asfy = 4.684(60,000\text{psi}) = 281,040#$

4. Find if the flange can handle the compressive force:
   $NCf = (.85f'c)(b)(hf) = .85(3000)(60)(5) = 765,000 > 281,040 \ldots$ the compression is handled by the flange and the analysis is the same as for a rectangular beam with a width $b = 60''$.

5. Find $a = Asfy / .85f'c b = 4.684(60,000) / .85(4000)(60) = 1.378''$

6. $c = a / .85 = 1.378 / .85 = 1.62$

$\epsilon_t = .003(21 – 1.62)/1.62 = 0.0359 \ldots$ therefore tension controls yielding and $\Phi = 0.9$.

7. $\Phi Mn = \Phi Asfy(d – a/2) = 0.9(4.684)(60\text{ksi})(21 – 1.378/2) = 5137.38 \text{k-in} = 428.12 \text{k-ft}$

27-6: Find the $\Phi$ Mn for the T-beam shown below. $f'c = 4\text{ksi}$ and $f_y = 60\text{ksi}$. The span is 24ft and the center-to-center beam spacing is 8'.

1. Find Effective Flange Length:
   $L/4 = 24(12)/4 = 72''$
   bw + 16 hf = 15 + 64 = 79''
   beam spacing = 96''

   use $b = 72''$

2. Check As min = .0033bwd = .0033(15)(22) = 1.089 < 4.0 (4-#9)...okay

3. Assume the steel yields. Find NT:
   $NT = Asfy = 4(60,000\text{psi}) = 240,000#$

4. Find if the flange can handle the compressive force:
   $NCf = (.85f'c)(b)(hf) = .85(4000)(72)(4) = 979,200 > 240,000 \ldots$ the compression is handled by the flange and the analysis is the same as for a rectangular beam with a width $b = 60''$.

5. Find $a = Asfy / .85f'c b = 4(60,000) / .85(4000)(72) = 0.98''$

6. $c = a / .85 = .98 / .85 = 1.15$

$\epsilon_t = .003(22 – 1.15)/1.15 = 0.054 > .005$ therefore tension controls yielding and $\Phi = 0.9$.

7. $\Phi Mn = \Phi Asfy(d – a/2) = 0.9(4)(60\text{ksi})(22 – .98/2) = 4646.16 \text{k-in} = 387.18 \text{k-ft}$
27-7:  Design reinforcement for a T-beam with $f'c = 4$ksi, $fy = 60$ksi, $bw = 14"$, $h = 27"$, $hf = 4"$ beam spacing = $7'$, beam span = $18'$ and $Mu = 250k$-f.

1. $Mu = 250k$-f = $3000k$-in
2. $d = 27 - 3 = 24"$, Assume $\Phi = 0.9$
3. $b \leq L/4 = 18(12)/4 = 54"$
   $b \leq bw + 16hf = 14 + 16(4) = 78"$
   $b \leq beam$ spacing = $84"$
   use $b = 54"$
4. $Mnf = .85f'cbhf(d - hf/2) = .85(4)(54')(4')(23 - 4/2) = 15,422.4k$-in
5. $0.9(17,136) = 15,422.4k$-in > $Mu = 3000k$-in  ...  Rectangular T-beam
6. $As = [.85f'cbd/fy][1 - \sqrt{1 - 2Mu/\Phi.85f'cbd^2}] = [.85(4)(54)(24)/60][1 - \sqrt{1 - 2(3000)/(.9(.85)(4)(54)(24)^2)]} = 2.35$in$^2$
7. Check that $As \geq As \min = bw_d(3\sqrt{f'c})/fy \geq 200bw_d/fy = 14(24)(3)(\sqrt{4000})/60000 = 1.06 \geq 200(14)(24)/60000 = 1.12$  ...  $As \min = 1.12 < 2.35$... okay
8. Use 3- #8 $As = 2.356$  $d = 27 - 1.5 - .375 - 1/2 = 24.625 > 24$ ... okay
9. $a = fyAs/(.85f'cb) = 60(2.356)/(.85(4)(54)) = 0.77$  $c = a/\beta1 = 0.77/.85 = 0.91$
10. Check $\epsilon_t = .003(d - c)/c = .003(24.625 - .91)/.91 = 0.081 > 0.004$.
11. $0.081 > 0.005$  ...  $\Phi = 0.9$
Answer: Use 3 - #8

27-8:  Design reinforcement for a T-beam with $f'c = 4$ksi, $fy = 60$ksi, $bw = 16"$, $h = 27"$, $hf = 3"$ beam spacing = $5'$, beam span = $22'$ and $Mu = 300k$-f.

1. $Mu = 300k$-f = $3600k$-in
2. $d = 27 - 3 = 24"$, Assume $\Phi = 0.9$
3. $b \leq L/4 = 22(12)/4 = 66"$
   $b \leq bw + 16hf = 16 + 16(3) = 64"$
   $b \leq beam$ spacing = $60"$
   use $b = 60"$
4. $Mnf = .85f'cbhf(d - hf/2) = .85(4)(60')(3')(24 - 3/2) = 13,770k$-in
5. $0.9(13,770) = 12,393k$-in > $Mu = 3600k$-in  ...  Rectangular T-beam
6. $As = [.85f'cbd/fy][1 - \sqrt{1 - 2Mu/\Phi.85f'cbd^2}]
   = [.85(4)(60)(24)/60][1 - \sqrt{1 - 2(3600)/(.9(.85)(4)(60)(24)^2)]} = 2.83$in$^2$
7. Check that $As \geq As \min = bw_d(3\sqrt{f'c})/fy \geq 200bw_d/fy$
   $= 16(24)(3)(\sqrt{4000})/60000 = 1.21 \geq 200(16)(24)/60000 = 1.28$  ...  $As \min = 1.28 < 2.83$ ... okay
8. Use 3- #9 $As = 3.0$  $d = 27 - 1.5 - .375 - 1.128/2 = 24.56 > 24$ ... okay
9. $a = fyAs/(.85f'cb) = 60(3)/(.85(4)(60)) = 0.0882$  $c = a/\beta1 = 0.882/.85 = 1.04$
10. Check $\epsilon_t = .003(d - c)/c = .003(24.56 - 1.04)/1.04 = 0.068 > 0.004$.
11. $0.068 > 0.005$  ...  $\Phi = 0.9$
Answer: Use 3 - #9
27-9: Design reinforcement for the inverted T-beam shown below. \( f'c = 4 \text{ksi}, \) \( f_y = 60 \text{ksi}, \) beam span = 20’ and \( \mu = 200 \text{k-ft}. \)

1. Determine \( \mu = 200 \text{k-ft} = 2400 \text{k-in} \)
2. Assume \( d = 28 - 3 = 25”, \) & \( \Phi = 0.9 \)
3. \( b = 6” \)
4. \( M_{nf} = 0.85 f'c bh f(d - hf/2) = 0.85(4)(6)(4)(27 - 4/2) = 2040 \text{k-in} \)
5. If \( \Phi M_{nf} = 0.9(2040) = 1836 \text{k-in} < \mu = 2400 \text{k-in} \) … go to step 12
12. \( Z_f = d - hf/2 = 25 - 4/2 = 23” \)
13. \( A_{sf} = M_{nf}/f_y Z_f = 2040/60/23” = 1.47 \text{in}^2 \)
14. \( d_w = d - hf = 25 - 4 = 21” \)
15. \( M_{nw} = (\mu - \Phi M_{nf})/ \Phi = (2400 - 1836)/0.9 = 626.67 \text{k-in} \)
16. \( a_w = d_w + \sqrt{d_w^2 - 2M_{nw}/(0.85 f'c b w)} = 21 + \sqrt{441 - 2(626.67)/(0.85(4)(22))} = 0.403 \text{in} \)
17. \( A_{sw} = 0.85 f'c a_w b_w / f_y = 0.85(4)(0.403)(22)/60 = 0.502 \text{in}^2 \)
18. \( A_s = A_{sf} + A_{sw} = 1.47 + 0.502 = 1.972 \text{ Use 2 - #9} \)
19. Calculate actual value of d: \( d = 28” - 1.5” - .375” - 1.128”/2 = 25.56” > 25” \) … okay
20. \( A_s = b_w d (3 \sqrt{f'c})/ f_y = 24(25.56)(3)(\sqrt{4000})/60000 = 1.778 \geq 200 b_w d / f_y = 200(22)(25.56)/60000 = 1.874 \) … As min = 1.874 < 2.0 … okay
21. \( \phi = a_w + hf = 4.403”, \) \( c = \alpha/\beta_1 = 4.403/0.85 = 5.18”. \)
   \( \epsilon_t = .003(d - c)/c = .003(25.56 - 5.18)/5.18 = 0.0118 > 0.005 \) … \( \Phi = 0.9, \)

Answer: Use 2 - #9

27-10: Design reinforcement for the box beam shown below. \( f'c = 4 \text{ksi}, \) \( f_y = 60 \text{ksi}, \) beam span = 20’ and \( \mu = 300 \text{k-ft}. \) NOTE: all sides are 4” thick

1. Determine \( \mu = 300 \text{k-ft} = 3600 \text{k-in} \)
2. Assume \( d = 36 - 3 = 33”, \) & \( \Phi = 0.9 \)
3. \( b = 24” \)
4. \( M_{nf} = 0.85 f'c b h f(d - hf/2) = 0.85(4)(24)(4)(33 - 4/2) = 10,118.4 \text{k-in} \)
5. If \( \Phi M_{nf} = 0.9(10,118.4) = 9106.56 \text{k-in} \) > \( \mu = 3600 \text{k-in} \) … Rectangular T-beam
6. \( A_s = [0.85 f'c b d / f_y] \left[ 1 - \sqrt{1 - 2 \mu/\Phi f'c b d^2} \right] \)
   \( = [.85(4)(24)(33)/60][1 - \sqrt{1 - 2(3600)/(0.9(4)(24)(33)^2)}] = 3.6 \text{in}^2 \)
7. \( A_{sw} = b_w d (3 \sqrt{f'c})/ f_y = 24(33)(3)(\sqrt{4000})/60000 = 2.5 \geq 200 b_w d / f_y = 200(24)(33)/60000 = 2.64 \) As min = 2.64 < 3.6 ... okay
8. Use 5 - #8 \( A_s = 3.927 \) \( d = 36 - 1.5 - .375 - 1/2 = 33.625 > 33 \) ... okay
9. \( a = f_y A_s / (0.85 f'c b) = 60(3.927)/(0.85(4)(24)) = 2.89 \) \( c = \alpha/\beta_1 = 2.89 / 0.85 = 3.40 \)
10. Check \( \epsilon_t = .003(d - c)/c = .003(33.625 - 3.4)/3.4 = 0.0267 > 0.004. \)
11. \( 0.0267 > 0.005 \) ... \( \Phi = 0.9 \)

Answer: Use 5 - #8
28-1 a: Design shear reinforcement for the concrete beams shown below.
Assume \( f'c = 4\text{ksi} \) and \( f_y = 60\text{ksi} \)

1. \( w_{BM} = \frac{.15(12)(18)}{144} = 0.225 \)
   \( w_u = 1.2(0.225) + 1.2 = 1.47k/f \)
   \( V_u = 1.47(16)/2 - 1.47(18/12) = 9.56k \)
2. \( \Phi Vc = \Phi^2(\sqrt{f'c})bwd \)
   \( = .75(2)(\sqrt{4000})(12)(18)/1000 = 20.49 \)
3. \( Is\ V_u \geq \Phi Vc/2? \quad 9.56 < 20.49/2 = 10.25 \)
   no shear reinforcement required.

28-1 b: repeat 28-1a with \( w_u = 6k/ft \)

1. \( w_{BM} = \frac{.15(12)(18)}{144} = 0.225 \)
   \( w_u = 1.2(0.225) + 6 = 6.27k/f \)
   \( V_u = 6.27(16)/2 - 6.27(18/12) = 40.76k \)
2. \( \Phi Vc = \Phi^2(\sqrt{f'c})bwd = .75(2)(\sqrt{4000})(12)(18)/1000 = 20.49k \)
3. \( Is\ V_u \geq \Phi Vc/2? \quad 40.76 > 20.49/2 = 10.25 \quad \text{go to step 4.} \)
4. \( Is\ \Phi Vc/2 \leq V_u \leq \Phi Vc? \quad no,\ 10.25 < 40.76 > 20.49 \quad \text{go to step 5} \)
5. \( If\ V_u > \Phi Vc, \text{calculate } Vs = (V_u - \Phi Vc)/\Phi = (40.76 - 20.49)/.75 = 13.44k \)
6. \( \text{Check that } Vs \leq 8(\sqrt{f'c})bwd: \quad 40.76 < 8(\sqrt{4000})(12)(18)/1000 = 109.29 \quad \text{okay} \)
7. \( \text{Assume } #3 \text{ stirrup, } s \leq Avfy/d/Vs = .22(60)(18)/13.44 = 17.68" \)
8. \( \text{Check spacing for minimum steel requirement:} \)
   \( \text{smax = Avfyt/50bw = .22(60000)/(50(12)) = 22"} \)
9. \( \text{Check ACI 11.5.4 maximum spacing requirement:} \)
   \( Vs = 13.44 < 4(\sqrt{4000})(12)(18)/1000 = 54.65 \)
   \( \text{if } Vs < 4(\sqrt{f'c})bwd \quad \text{smax} \leq d/2 \leq 24" = 18/2 = 9" < 17.68 \quad \text{use 9"}} \)
10. \( \text{Check Minimum spacing } s_{\text{min}} = 4" < 9" \quad \text{okay} \)
11. \( \text{Locate where } \Phi Vc \text{ and } \Phi Vc/2 \text{ are located on shear diagram in terms of } x. \)
    \( \Phi Vc \text{ is @ x where } 16'(6.27k/l)/2 - 6.27x = 20.49 \)
    \( 50.16 - 20.49 = 6.27x \quad \text{... } x = 4.73' \quad (\text{and } 16'-4.73' = 11.27') \)
    \( \Phi Vc/2 \text{ is @ x where } 16'(6.27)/2 - 6.27x = 10.25 \quad x = 6.37' \quad (\text{and } 16'-6.37' = 9.63') \)
12. \( \text{Indicate what shear reinforcement is required and where.} \)
    \( \text{Zone where } V > \Phi Vc \text{ use leer of values from steps 7 and 9:} \)
    \( \text{Use #3 stirrups @ 9" } \quad 0 < x < 4.73' \quad \text{& } 11.27' < x < 16' \)
    \( \text{Zone where } \Phi Vc/2 < V < \Phi Vc \text{ use lesser of values from steps 8 and 9:} \)
    \( \text{Use #3 stirrups @ 9" } \quad 4.73 < x < 6.37' \quad \text{& } 9.63' < x < 11.27' \)
28-2 a: Design shear reinforcement for the concrete beams shown below. Assume $f'c = 4\text{ksi}$ and $fy = 60\text{ksi}$

1. $w_{BM} = .15(14)(17)/144 = 0.248$
2. $w_u = 1.2(0.248) = 0.298k/f$
3. $Vu = 0.298(24)/2 -.298(17/12) + 4.5 = 7.65k$
4. $\Phi Vc = \Phi 2(\sqrt{f'c})bwd$
   = $0.75(2)(\sqrt{4000})(14)(17)/1000 = 22.58k$
5. Is $Vu \geq \Phi Vc/2$? $7.65 < 22.58/2 = 11.29$
   no shear reinforcement required.

28-2 b: Repeat 28-1a with $Pu = 15k$

1. $w_{BM} = .15(14)(17)/144 = 0.248$
2. $w_u = 1.2(0.248) = 0.298k/f$
3. $Vu = 0.298(24)/2 -.298(17/12) + 1.5(15) = 25.65k$
4. $\Phi Vc = \Phi 2(\sqrt{f'c})bwd$
   = $0.75(2)(\sqrt{4000})(14)(17)/1000 = 22.58k$
5. Is $Vu \geq \Phi Vc/2$? $25.65 > 22.58/2 = 11.293$
   go to step 4.
6. Is $\Phi Vc/2 \leq Vu \leq \Phi Vc$? $11.293 < 25.65 > 22.58$ go to step 5
7. If $Vu > \Phi Vc$, calculate $Vs = (Vu - \Phi Vc)/\Phi = (25.65 - 22.58)/.75 = 4.09k$
8. Check that $Vs \leq 8(\sqrt{f'c})bwd$: $4.09 < 8(\sqrt{4000})(14)(17)/1000 = 120.42$ ... okay
9. Assume #3 stirrups, $s \leq Avfy_d/Vs = .22(60)(17)/4.09 = 54.87''$
10. Check spacing for minimum steel requirement:
    $s_{max} = Avfyt/50bw = .22(60000)/(50(14)) = 18.86 < 54.87''$ ... $s_{max} = 18.86''$
11. Locate where $\Phi Vc$ and $\Phi Vc/2$ are located on shear diagram in terms of x.
12. Indicate what shear reinforcement is required and where.
    Zone where $V > \Phi Vc$ use lesser of values from steps 7 and 9:
    Zone where $\Phi Vc/2 < V < \Phi Vc$ use lesser of values from steps 8 and 9:
    Use #3 stirrups @ 8.5'' $0 < x < 6'$ & $18' < x < 24''$ [Zone where $V > \Phi Vc$]
28-3a: Design shear reinforcement for the concrete beams shown below. Assume $f'c = 4\text{ksi}$ and $f_y = 60\text{ksi}$

1. $w_{bm} = 0.15(12)(20)/144 = 0.25$
2. $w_u = 1.2(2.5) + 1.2(2) = 2.7k/ft$
3. $Vu = 2.7(6) - 2.7(20/12) = 11.7k$
4. $\Phi Vc = \Phi 2(\sqrt{f'c})bwd$
5. $= 0.75(2)(\sqrt{4000})(12)(20)/1000 = 22.77$
6. Is $Vu \geq \Phi Vc/2$? $11.7 > 22.77/2 = 11.385$

28-3b: repeat 28-1a with $w_u = 10k/ft$

1. $w_{bm} = 0.15(12)(20)/144 = 0.25$
2. $w_u = 1.2(2.5) + 1.2(2) = 2.3k/ft$
3. $Vu = 2.3(6) - 2.3(20/12) = 9.97k$
4. Is $Vu \geq \Phi Vc/2$? $9.97 > 22.77/2 = 11.385$
5. Is $\Phi Vc/2 \leq Vu \leq \Phi Vc$? no, $9.97 < 22.77$ go to step 5
6. Check that $Vs \leq 8(\sqrt{f'c})bwd$: $9.97 < 8(\sqrt{4000})(12)(18)/1000 = 109.29$ okay
7. Assume #3 stirrup, $s \leq Avfyt/Vs = 0.22(60)(18)/13.44 = 17.68"$
8. Check spacing for minimum steel requirement: $s_{max} = Avfyt/50bw = 0.22(60000)/(50(12)) = 22 > 17.68"$ okay
9. Check ACI 11.5.4 maximum spacing requirement: $Vs = 13.44 < 4(\sqrt{4000})(12)(18)/1000 = 54.65$
   if $Vs < 4(\sqrt{f'c})bwd$ ... $s_{max} \leq d/2 \leq 24"/18/2 = 9" < 17.68$ ... use 9"
10. Check Minimum spacing $s_{min} = 4" < 9"$ okay
11. Locate where $\Phi Vc$ and $\Phi Vc/2$ are located on shear diagram in terms of $x$.
   $\Phi Vc$ is @ $x$ where $16'(6.27k/ft)/2 - 6.27x = 20.49$
   $50.16-20.49 = 6.27x \ldots x = 4.73' \ldots 16'-4.73' = 11.27'$
   $\Phi Vc/2$ is @ $x$ where $16(6.27)/2 - 6.27x = 10.25 \ldots x = 6.37' \ldots 16'-6.37' = 9.63'$
12. Indicate what shear reinforcement is required and where.
   Zone where $V > \Phi Vc$ use lesser of values from steps 7 and 9:
   Use #3 stirrups @ 9" $0 < x < 4.73' \& 11.27'< x < 16' \ [\text{Zone where } V > \Phi Vc]$
   Zone where $\Phi Vc/2 < V < \Phi Vc$ use lesser of values from steps 8 and 9:
   Use #3 stirrups @ 9" $4.73 < x < 6.37' \& 9.63'< x < 11.27' \ [\text{Zone where } \Phi Vc/2 < V < \Phi Vc]$
28-4 Find the immediate and long term deflections of the concrete beam shown below. Assume $f'c=4$ksi and $fy = 60$ksi & $h = d+3''$

1. $n = \frac{E_s}{E_c} = \frac{29000}{57\sqrt{4000}} = 8.04$
2. $y = \frac{nAs[\sqrt{(1 + 2bd/nAs)} - 1]}{b}$
   $= 8.04[3]\sqrt{(1 + 2(12)(18)/8.04(3)) - 1]}/12 = 6.73''$
3. $I_{cr} = \frac{by}{3} + \frac{nAs(d - y)^2}{b}$
   $= 12(6.73)^3/3 + 8.04(3)(18 - 6.73)^2 = 4282.84$
4. $I_g = \frac{12(21)^3}{12} = 9261$
   $fr = \frac{(7.5\sqrt{4000})}{1000} = 0.474ksi$
   $M_{cr} = frI_g/y_t$
   $= 0.474ksi (9261) / (21/2) = 418.07k-in$
   $w = \frac{(150)(12/12)(21/12)}{1000#v/k + (1.2)} = 1.46k/f$
   $M_a = 1.46(16)^2(12)/8 = 560.65k-in$
   $M_{cr}/M_a = 418.07/560.65 = 0.746$
9. $I_e = \left\{\frac{M_{cr}}{M_a}\right\}I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr} = \left\{\frac{.746}{1}(9261) + \left[1 - (.746)^3\right]\right\}4282.84 = 6349.58in^4$
10. $\Delta_i = 5wL^4(1728)/384EI = 5(1.46k/f)(16)^4(1728in^3/f^3)/[384(57 \sqrt{4000})(6349.58)] = 0.107''$
11. $\zeta = 2.0$ for 5 years or more.
12. $w_{sustained} = w = 1.46k/f$
13. $\Delta_i = 0.107''$
14. $\rho' = 0$
15. $\Delta LT = \Delta i \xi (1 + 50\rho') = 0.107(2)/(1 + 0) = 0.214''$
16. Total deflection = $\Delta = 0.107 + 0.214 = 0.321''$

28-5 Find the immediate and long term deflections of the concrete beam shown below. Assume $f'c=4$ksi and $fy = 60$ksi & $h = d+3''$

1. $n = \frac{E_s}{E_c} = \frac{29000}{57\sqrt{4000}} = 8.04$
2. $y = \frac{nAs[\sqrt{(1 + 2bd/nAs)} - 1]}{b}$
   $= 8.04[3.142]\sqrt{(1 + 2(12)(20)/8.04(3.142)) - 1]}/12 = 7.27''$
3. $I_{cr} = \frac{by}{3} + \frac{nAs(d - y)^2}{b}$
   $= 12(7.27)^3/3 + 8.04(3.142)(20 - 7.27)^2 = 5630.69$
4. $I_g = \frac{12(23)^3}{12} = 12167$
   $fr = \frac{(7.5\sqrt{4000})}{1000} = 0.474ksi$
   $M_{cr} = frI_g/y_t$
   $= 0.474ksi (12167) / (23/2) = 501.49k-in$
   $w = \frac{(150)(12/12)(23/12)}{1000#v/k + (2)} = 2.29k/f$
   $M_a = 2.29(6)^2(12)/8 = 494.64k-in$
   $M_{cr}/M_a = 501.49/494.64 = 1.014$
9. $I_e = \left\{\frac{M_{cr}}{M_a}\right\}I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr} = \left\{\frac{1.014}{1}(12167) + \left[1 - (1.014)^3\right]\right\}5630.69 = 12445.39in^4$
10. $\Delta_{max} = wL^4(1728)/8EI = (2.29k/f)(6)^4(1728in^3/f^3)/[8(57 \sqrt{4000})(12445.39)] = 0.014''$
11. $\zeta = 2.0$ for 5 years or more.
12. $w_{sustained} = w = 2.29k/f$
13. $\Delta_i = 0.014''$
14. $\rho' = 0$
28-6 Find the immediate and long term deflections of the concrete beam shown below. Assume $f'c=4$ksi and $f_y = 60$ksi & $h = d+3”$

1. $n = \frac{E_s}{E_c} = \frac{29000}{\sqrt{57 \sqrt{4000}}} = 8.04$
2. $y = \frac{nA_s}{\sqrt{[1 + 2bd/nA_s] - 1}}/b$
   
   $= 8.04(3.927)\left[\sqrt{1 + 2(12)(18)/8.04(3.927)} - 1\right]/12 = 7.45”$
3. $I_{cr} = \frac{by^3}{3} + nA_s(d-y)^2$
   
   $= 12(7.45)^3/3 + 8.04(3.927)(18 - 7.45)^2 = 5168.14$
4. $I_g = 12(21)^3/12 = 9261$

   $f_r = \frac{7.5\sqrt{4000}}{1000} = 0.474$ksi

   $M_{cr} = f_rI_g/y_i = 0.474$ksi, $9261)/(21/2) = 418.07$-k-in

   $w = (150)(12/12)/(21/12)/1000#k + (1) = 1.26k/f$; $P = 2k$

   $M_a = 1.26(20)^2(12)/8 + 2(20)(12)/4 = 876$-k-in,

   $M_{cr}/M_a = 418.07/876 = 0.477$
9. $I_e = \left\{M_{cr}/M_a\right\}^3I_g + \left[1 - \left(M_{cr}/M_a\right)^3\right]I_{cr} = \{0.477\}^3(9261) + \left[1 - (0.477)^3\right](5168.14) = 5612.34$-in$^4$

10. $\Delta_i = 5wL^4(1728)/384EI + PL^3/48EI = 5(1.26k/f)(20)^4(1728)/[384(57 \sqrt{4000})(5612.34)]$

    + $2(20)^3(1728)/[48(57 \sqrt{4000})(5612.34)] = 0.25”$
11. $\xi = 2.0$ for 5 years or more.
12. $w_{sustained} = w = 1.26k/f$
13. $\Delta_i = 0.25”$
14. $\rho' = 0$
15. $\Delta_{LT} = \Delta_i\xi/(1 + 50\rho’) = .25(2)/(1 + 0) = 0.5”$
16. Total deflection $= \Delta = 0.25 + .25 = 0.75”$
29-1: Find allowable axial load on a 12X12" tied column with a maximum unbraced length of 14’, f’c = 4ksi, fy = 60ksi with 8 - #8 longitudinal bars.

From Table A.4.1, 8 - #8s have an area of steel = Ast = 6.283in²
Ag = 12² = 144in²
ΦPn = .8(.65)[.85f’c(Ag – Ast) + fy(Ast)] = .8(.65)[.85(4)(144 – 6.283) + 60(6.283)] = 439.51k

29-2: Check the adequacy of a short 24" X 24" tied column with a 1.5" cover, f’c = 4ksi, fy = 60ksi, 16-#10 and Pu = 1600k. The column ties are #3 bars at 17” o.c.

1. Check that 0.01 < ρg = Ast/Ag < 0.08.
   For 16-#10 from Table A.4.1, Ast = 16.0in²
   Ag = 24² = 576in²
   ρg = Ast/Ag = 20.268/576 = .035
   0.01 < ρg = .035 < 0.08, therefore the column is adequate for ρg.
2. From Table A.4.3, 16 - #10 will fit in a 24" square column and there are greater than 4 bars, therefore column is adequate for steel placement.
3. Check that Pu < ΦPn. Pu = 1200k
   ΦPn = .65(8)[.85f’c(Ag – Ast) + fy(Ast)] = .65(8)[.85(4)(576 – 20.268) + 60(20.268)] = 1614.9k
   Pu = 1600 < 1614.9 = ΦPn, therefore column is adequate for load.
4. Check tie size: Okay for minimum #3 when size of bars < #11
   Tie spacing criteria:
   16d₅₉ = 16(1.27) = 20.32”
   48d₅₉ = 48(.375) = 18”
   least column dimension = 24”
   Use 18” … okay for tie spacing
5. Check clear spacing between longitudinal bars on one face =
   \[(24 – 3 – 2(.375) – 5(1.27))/4 = 3.475" < 6"\] therefore column is adequate for longitudinal bar spacing.
29-3: Design a short square column to carry a dead load of 500k and a live load of 800k.

1. Use $f'c = 4$ksi, $fy = 60$ksi, $\rho_g = .03$
2. $Pu = 1.2(500) + 1.6(800) = 1880k$
3. $Ag = Pu / (.65(0.8)[.85f'c(1 – \rho_g) + fy\rho_g]) = 1880 / (.65(0.8)[.85(4)(.97) + 60(.03)]) = 709.18in^2$
4. $\sqrt{709.18} = 26.63"$ round up to next whole inch. Use 27 X 27" column $Ag = 729in^2$
5. $\Phi_Pc = .65(0.8)Ag[.85f'c(1 – \rho_g)] = .65(0.8)(729)(.85)(4)(.97) = 1250.21k$
6. Determine load on steel $\Phi_Ps = Pu – \Phi_Pc = 1880 – 1250.21 = 629.79k$
7. $Ast = \Phi_Ps / (.85) fy = 629.79 / .85 / 60 = 20.19in^2$
8. From Table A.4.1, the area of 16-#10 = 20.268 > 20.19 and this is a multiple of 4 (required for even distribution in a square column). From Table A.4.3, side width = 27", 16 - #10 will fit.
9. Use #5 ties $< 48(.625) = 30$ or $16(1.27) = 20.32$ or $28" ... s = 20.32" ... round down to 20"
10. Check clear spacing between longitudinal bars on one face = $(h – 2(cover) – 2dtie – (#bars/4 + 1)db)/(#bars/4) = (27 – 3 – 2(.625) – 5(1.27))/4 = 4.1" < 6"$ therefore no additional ties are required.

29-4: Design a short round column with spiral reinforcement to carry a dead load of 500k and a live load of 800k.

1. Use $f'c = 4$ksi, $fy = 60$ksi, $\rho_g = .03$
2. $Pu = 1.2(500) + 1.6(800) = 1880k$
3. $Ag = Pu / (.75(0.85)[.85f'c(1 – \rho_g) + fy\rho_g]) = 1800 / (.75(0.85)[.85(4)(.97) + 60(.03)]) = 578.47 = \pi h^2/4$
4. $h = \sqrt{(578.47(4))/\pi} = 27.14'$ Round up to next whole number and use 28" dia. column $Ag = \pi 28^2/4 = 615.75in^2$
5. $\Phi_Pc = .75(0.85)Ag[.85f'c(1 – \rho_g)] = .75(0.85)(615.75)(.85)(4)(.97) = 1294.6k$
6. $\Phi_Ps = Pu – \Phi_Pc = 1880 – 1294.6 = 585.4k$
7. $Ast = \Phi_Ps / (.75) fy = 585.4 / .85 / 60 = 15.3in^2$
8. From Table A.4.1, choose 12 - #11 = 19.7in^2 From Table A.4.3, 12 - #11 will fit. $d_{ch} = 28-3 = 25"$
9. Using a 5/8" diameter spiral, $A_{ch} = \pi d_{ch}^2/4 = 452.4in^2$
   $ps_{min} = .45((Ag/Ach) – 1)(f'c/fytl) = .45((615.75/452.4) – 1)(4/60) = 0.0108$
   $s_{max} = 4Asp/dchps = 4(.31)/25(.0108) = 4.59"$
   $1 + dsp < s < 3 + dsp$ ... $s < 3 + 0.625 = 3.625"$
Use $s = 3.5"$
29-5: Find the practical nominal moment for the column shown below. 8 - #8 bars, f'c = 4ksi and fy = 60ksi, e = 3".

1. c.c. bars = 20 - 2(2.5) = 15", h = 20"  γ = 15/20 = 0.75  use diagram A.4.5.3
2. ρg = Ast/Ag = 6.283/20(20) = 0.016
3. Locate ρg = 0.016 drawn as heavy curve below.
4. slope = h/e = 20/3 = 6.67 = 1/.15
   Drawn from origin as heavy line.
5. Find the intersection of the ρg curve and the h/e line from steps 3 and 4. Draw a horizontal line through the intersection to locate Kn = .74, and a vertical line through the intersection to locate Rn = .115
6. Determine Φ by checking strain. The point of intersection is above 1.0 line (radial line) for fs/fy, therefore the column steel is in compression and Φ = 0.65.
7. ΦPn = ΦKn f'cAg = .65(.74)(4)(20)(20) = 769.6k
   ΦMn = ΦRnf'cAgh = .65(.115)(4)(400)(20)/12in/f = 199.33k-f
   Or ΦMn = ΦPne = 769.6(3)/12 = 192.4k-f

ΦMn = 192.4k-f
29-6: Find the practical nominal moment for the column shown below. 11 - #8 bars, f’c = 4ksi and fy = 60ksi, e = 6”.

1. c.c. bars = 20-2(2.5)= 15”, h = 20” ... γ = 15/20 = 0.75 ...
   use diagram C4-60.8 from figure A4.5
2. ρg = Ast/A_g = 11(.785)/(π20^2/4) = 0.275
3. Locate ρ_g = 0.016 drawn as heavy curve below.
4. slope = h/e = 20/6 = 3.33= 1/3
   Drawn from origin as heavy line.
5. Find the intersection of the ρ_g curve and the h/e line from steps 3 and 4. Draw a horizontal line through the intersection to locate K_n = .46, and a vertical line through the intersection to locate R_n = .135
6. Determine Φ by checking strain. The point of intersection is above 1.0 line (radial line) for f_s/f_y, therefore the column steel is in compression and Φ = 0.65.
7. ΦP_n = ΦK_nf’cAg = .65(.46)(4)(π20^2/4) = 375.73k
   ΦM_n = ΦR_nf’cAgh = .65(.135)(4)(π20^2/4)(20)/12in/f = 183.78k-f
   Or ΦM_n = ΦP_ne = 375.73(6)/12 = 187.87k-f

ΦM_n = 183.78k-f
29-7: Design a round column with spiral reinforcement. Pu = 800kips, e = 6", f'c = 4ksi and fy = 60ksi.

1. Pu = 800k. Mu = Pu/e = 800k(6") = 4800k-in

2. \[ Ag = \frac{Pu}{0.85(0.75)(0.85f'c(0.99) + 0.01fy)} = \frac{800}{0.85(0.75)(0.85(4)(0.99) + 0.01(60))} = 256.04 \text{in}^2 \]

3. \[ h = \sqrt{\frac{256.04(4)}{\pi}} = 18.06" \] use \( h = 19" \). \[ Ag = \pi(19^2)/4 = 283.53 \text{in}^2 \]

4. Assume #9 size and a 3/8" spiral size.

5. c.c long. Bars \( = 19 - 3 - 2(0.375) - 1.128 = 15.25" \)

\[ \gamma = \frac{15.25}{19} = 0.8 \] use Table A.4.4.6

6. Required \( Kn = \frac{Pu}{\Phi f'cAg} = \frac{800}{0.75(4)(283.53)} = 0.94 \)

Required \( Rn = \frac{Mu}{\Phi fc'Agh} = \frac{4800}{0.75(4)(283.53)(19)} = 0.30 \)

Locate the point of intersection on the diagram.

7. \( \rho_g > 0.08 \) ... must increase size of column. \( d = 21" \), \( Ag = \pi(21^2)/4 = 346.36 \text{in}^2 \)

6A. Required \( Kn = \frac{Pu}{\Phi f'cAg} = \frac{800}{0.75(4)(346.36)} = 0.77 \)

Required \( Rn = \frac{Mu}{\Phi fc'Agh} = \frac{4800}{0.75(4)(346.36)(21)} = 0.22 \)

7. \( \rho_g = 0.058 \)

8. \( As = \rho_gAg = 0.058(346.36) = 20.09 \text{in}^2 \)

9. From Table A.4.2 and A.4.3, 14- #11 \( As = 2(10.93) = 21.86 \)

10B: For spiral columns, design spirals.

\[ d_{ch} = 21 - 3 = 18" \] & \( Ach = \pi(18^2)/4 = 254.47 \text{in}^2 \)

\[ ps \text{ required} = 0.45(Ag/Ach - 1)(fc'/fy) = 0.45(346.36/254.47 - 1)(4/60) = 0.0108 \]

\[ s = 4A_{sp}/d_{ch}ps = 4(0.11)/18(0.0108) = 2.26" \]

USE spiral spacing @ 2.25"

Clear spacing = 2.25 - 0.375 = 1.875 > 1" and < 3" ... okay.

ANSWER: 21" diameter column with 14 - #11 and 3/8" spiral at 2.25" o.c.
Design a square column with ties $Pu = 1500$ kips, $e = 10''$, $f'c = 4$ ksi and $fy = 60$ ksi.

1. $Pu = 1500$, $Mu = Pue = 1500(10'') = 15000$ k-in
2. Estimate the column size based on $\rho_g = 0.01$ and ignoring the eccentricity:
   
   $A_g = \frac{Pu}{0.8 \times 0.65 \times (0.85 f'c (0.99) + 0.01 fy)} = \frac{1500}{0.8 \times 0.65 \times (0.85 (4)(0.99) + 0.01 (60))} = 727.34 \text{ in}^2$

3. $h = \sqrt{727.34} = 26.97$ use 27'' $A_g = 27^2 = 729 \text{ in}^2$.

4. Assume #9 size and #4 tie.

5. $c.c$ long. Bars = 29 – 3 – 2(3.75) – 1.128 = 24.122''
   
   $\gamma = 24.122/29 = 0.83$ use diagram R4-60.8 from Figure A4.5

6. Required $Kn = \frac{Pu}{0.65 (4) (729)} = 0.79$
   
   Required $Rn = \frac{Mu}{0.65 (4) (729) (27)} = 0.293$

   Locate the point of intersection on the diagram.

7. $\rho_g = 0.068$ and $\Phi = 0.65$

8. $As = \rho_g A_g = 0.068 \times 729 = 49.57 \text{ in}^2$. This would require 32 - #11 which will not fit in a 27'' square column ... go larger. Let $h = 30$, $A = 900$.

6A. Required $Kn = \frac{Pu}{0.65 (4) (900)} = 0.64$
   
   Required $Rn = \frac{Mu}{0.65 (4) (900) (30)} = 0.22$

7A. $\rho_g = 0.0395$ and $\Phi = 0.65$

8A. $As = \rho_g A_g = 0.0395 \times 900 = 35.55 \text{ in}^2$

9. bars must be a multiple of 4. Therefore, use 24 - #11, $As = 12.492(3) = 37.48$. Checking with Table A.4.3 shows 24 - #11 are allowed in a 30'' square column.

10A. For tied columns, design ties.

Design ties:

- $dch = 30 – 3 = 27''$ & $Ach = 729$
- $s = \text{smallest of } 16(1.41) = 22.56'' \text{ or } 48(.5) = 24'' \text{ or } 30''$. Use #4 ties @22.5''

Check clear spacing of longitudinal bars:

- $(h – 2(\text{cover}) – 2\text{tie} – (\#\text{bars}/4 + 1)\text{db})/\#\text{bars}/4 = (30 – 3 – 2(.5) – 7(1.41))/6 = 2.68'' < 6''$

therefore no additional ties are required.
30-1: Find the development length for the reinforcement bars shown below. 
\( f'c = 4 \text{ksi} \), \( fy = 60 \text{ksi} \), uncoated bars, As req'd = 3.05in^2

1. Calculate \( K_D = \frac{3}{40} (fy/\sqrt{f'c}) = \frac{3}{40} (60000/\sqrt{4000}) = 71.15 \)
2. Determine \( \Psi_t, \Psi_e, \Psi_s, \) cb.
   \( \Psi_t = 1.0 \) (less than 12" below reinforcing steel)
   \( \Psi_e = 1.0 \) (uncoated & galvanized)
   \( \Psi_s = 1.0 \) (#7 and larger)
   cb:
   \[ \text{center to edge} = \frac{1}{2} + .375 + 1.5 = 2.375" \]
   \[ \frac{1}{2} \text{ center to center} = .5(14 - 2(1.5 + .375 + 1/2))/3 = 1.54" \]
   \( c_b = 1.54" \)
3. Assume \( Ktr = 0 \), find Calculate \( c_b/d_b = 1.54/1 = 1.54 < 2.5 \ldots \) use 1.54
4. Calculate \( K_{ER} = \) As required/As used = 3.05/3.142 = 0.971
5. \( L_d = K_{ER} K_D [\Psi_t \Psi_e \Psi_s/(c_b/d_b)](d_b) = 0.971(71.15)[1/1.54](1) = 44.86" \)

30-2: Find the development length for the reinforcement bars shown below. 
\( f'c = 4 \text{ksi} \), \( fy = 60 \text{ksi} \), epoxy coated, As req'd = 3.05in^2

1. Calculate \( K_D = \frac{3}{40} (fy/\sqrt{f'c}) = \frac{3}{40} (60000/\sqrt{4000}) = 71.15 \)
2. Determine \( \Psi_t, \Psi_e, \Psi_s, \) cb.
   \( \Psi_t = 1.0 \) (less than 12" below reinforcing steel)
   \( \Psi_e = 1.5 \) (epoxy)
   \( \Psi_s = 1.0 \) (#7 and larger)
   cb:
   \[ \text{center to edge} = 1.41/2 + .375 + 1.5 = 2.58" \]
   \[ \frac{1}{2} \text{ center to center} = .5(11 - 2(1.5 + .375 + 1.41/2))/1 = 2.92" \]
   \( c_b = 2.58" \)
3. Assume \( Ktr = 0 \), find Calculate \( c_b/d_b = 2.58/1.41 = 1.83 < 2.5 \ldots \) use 1.83
4. Calculate \( K_{ER} = \) As required/As used = 3.05/3.123 = 0.977
5. \( L_d = K_{ER} K_D [\Psi_t \Psi_e \Psi_s/(c_b/d_b)](d_b) = 0.977(71.15)[1/1.83](1.41) = 80.34" \)
30-3: Find the development length for the reinforcement bars shown below.

\[ f'c = 4\text{ksi}, \quad f_y = 60\text{ksi}, \quad \text{galvanized}, \quad \text{As req'd} = 7.34\text{in}^2 \]

1. Calculate \( K_D = \frac{(3/40)(f_y/\sqrt{f'c})}{(3/40)(60000)/(1(\sqrt{4000}))} = 71.15 \)
2. Determine \( \Psi_t, \Psi_e, \Psi_s, c_b \).
   \( \Psi_t = 1.0 \) (less than 12" below reinforcing steel)
   \( \Psi_e = 1.0 \) (galvanized)
   \( \Psi_s = 1.0 \) (#7 and larger)
3. \( c_b = \frac{1}{2} + .375 + 1.5 = 2.375'' \)
4. \( K_{ER} = \frac{\text{As required}}{\text{As used}} = 7.34/7.601 = 0.966 \)
5. \( L_d = K_{ER}K_D[\Psi_t\Psi_e\Psi_s/(c_b/d_b)][(d_b)] = 0.966(71.15)(1/1.97)(1.27) = 44.31'' \)

30-4: A simple beam 15" by 28" with a span of 24' three point loads, \( Pu = 50k \) at \( x = 6', 12' \) and 18'. The reinforcement is 6-#8 evenly spaced inside a #3 stirrups. \( f'c = 4\text{ksi}, \quad f_y = 60\text{ksi} \). Find at what point bars can be cut off.

1. \( w_u = 1.2[.15(15/12)(28/12)] = 0.34k/f, \quad Pu = 50k \)
2. \( M_u = 0.34(24)^2/8 + 50(24)/2 = 624.48k-f = 7493.76k-in \)
3. For 6 - #8, \( As = 4.712\text{in}^2 \)
4. \( As_{min} = .0033(15)(28 - 1.5 - .375 - 1/2) = 1.27\text{in}^2 \)
5. \( d = 28 - 1.5 - .375 - 1/2 = 25.625'' \)
6. \( a = f_yAs/(0.85f'cb) = 60(4.712)/(.85(4)(15))^2 = 9.74'' \)
7. \( M_n = f_yAs(d - a/2) = 60(4.712)(25.625 - 9.74/2) = 6461.56k-in = 538.46k-f \)
8. \( \Phi M_n = .9(538.46) = 484.26k-f = M_x \)
   \( M_x = 79.08x - .34x^2/2 - 50<x-6> - 50<x-12> - 50<x-18> \)
   \( x = 6.6' = 79.2'' \) where \( x \) is the distance from the support.
9. Check \( L_d \) to see if 2 bars can be cut at \( x = 6.6' \):
   \( K_D = (3/40)(f_y/\sqrt{f'c}) = (3/40)(60000)/(1(\sqrt{4000})) = 71.15 \)
   \( d_b = 1'' \)
   \( \Psi_t = 1.0, \quad \Psi_e = 1.0, \quad \Psi_s = 1.0 \)
   \( c_b = \frac{1}{2} + .375 + 1.5 = 2.375'' \)
   Or \( \frac{1}{2} \) center to center = .5(15 - 2(1.5 + .375 + 1.27/2))/2 = 2.05''
   \( c_b/d_b = 1.025/1 = 1.025 < 2.5 \) okay
   \( L_d = K_{ER}K_D[\Psi_t\Psi_e\Psi_s/(c_b/d_b)][(d_b)] = 1(71.15)(1/1.025)(1) = 69.76'' \)
   2 bars can be cut off at (24' - 69.76/12)/2 = 9.09'
7. The cutoff must also be closer to the support than the theoretical cutoff point by the larger of \( d \) or 12\( d_b \):
   \( d = 25.625'' = 2.14' \) and 12\( d_b = 12(1)/12 = 1' \).
   2.14' governs  
   \( x = 6.6' - 2.14' = 4.46' \)
8. Terminate 2 bars at 4.46' from each support.
9. Find the moment, \( \Phi M_n \) for 4 - #8 : \( As = 3.142\text{in}^2 \) > \( As_{min} = 1.27\text{in}^2 \)
\[ a = \frac{f_y A_s}{(0.85 f'_c b)} = \frac{60(3.142)}{(0.85(4)(15)} = 3.70'' \]

\[ M_n = f_y A_s (d - \frac{a}{2}) = 60(3.142)(25.625 - \frac{3.70}{2}) = 4482.06 \text{ k-in} = 373.51 \text{ k-f} \]

\[ \Phi M_n = 0.9(373.51) = 336.15 \text{ k-f} \]

\[ x = 4.29'' = 51.48'' \text{ where } x \text{ is the distance from the support.} \]

**6A.** Check the development length to see if 4 bars can be cut at \( x = 4.29'' \):

\[ K_D = \frac{3}{40}(\frac{f_y}{\lambda \sqrt{f'_c}}) = \frac{3}{40}(\frac{60000}{1(\sqrt{4000})}) = 71.15 \]

\[ d_b = 1'' \quad \Psi_t = 1.0, \quad \Psi_e = 1.0, \quad \Psi_s = 1.0 \]

\[ c_b = \text{lesser of}: \]

- center to edge = \( 1/2 + .375 + 1.5 = 2.375 \)
- Or \( 1/2 \) center to center = \( .5(15 - 2(1.5 + .375 + 2.05 + 1/2)) = 3.075'' \)

\[ c_b/d_b = 2.375/1 = 2.375 < 2.5 \text{ okay} \]

\[ L_d = K_D \Psi_t \Psi_e \Psi_s / (c_b/d_b) = 1(71.15)(1)(1)/(2.375)(1) = 29.96'' \]

4 bars can be cut off at \( (24' - 29.96/12)/2 = 10.75' \)

**7A.** The cutoff must also be closer to the support than the theoretical cutoff point by the larger of \( d \) or \( 12d_b \):

\[ d = 25.625'' = 2.14' \quad \text{and} \quad 12d_b = 12(1)/12 = 1'. \]

2.14' governs \( \ldots \) \( x = 4.29'' - 2.14'' = 2.15'' \)

**8A.** Terminate an additional 2 bars at 2.15' from each support.

**9A.** Find the moment, \( \Phi M_n \) for 2 - #8:

\[ A_s = 1.571 \text{ in}^2 > A_s \text{ min} = 1.27 \text{ in}^2 \quad \ldots \quad \text{at least 2-#8's are required for minimum } A_s. \]
31-1: Design a reinforced concrete bearing wall to support 16" steel beams spaced at 8' o.c. The beams bear on the full thickness of the wall with \( b_f = 14.5" \). The bottom and top of the wall are a fixed connections. The wall is 18' high and the load from each beam, \( P_u = 40k \). \( f_c = 4\text{ksi}, \ f_y = 60\text{ksi} \).

1. \( b_{min} = h/25 = (1/25)(18)(12) = 8.64" \quad \text{...} \quad b = 9" \)
2. Bearing strength of concrete = \( \Phi(.85f_c)(b)(b_f) = 0.65(.85)(4)(9)(14.5) = 288.41k \)
   \( 288.41k > 40k = P_u \quad \text{... okay.} \)
3. Effective length of wall is lesser of: distance between loads = 8' = 96" or width of bearing + 4b = 14.5 + 4(9) = 50.5" Use 50.5" \( \text{... Le = 50.5"} \)
4. \( \Phi P_n = 0.55\Phi f_c A_g[1 − (kh/32b)^2] = .55(.65)(4)(9)(50.5)[1 − (.65(18)(12)/32/9)^2] = 495.47k > 40k \quad \text{okay} \)
5. Reinforcing steel: (assume #5 or smaller)
   - Vertical steel: \( A_s = .0144(9) = .130 \) use #4 @18"
   - Horizontal Steel: \( A_s = .024(9) = .216 \) use #4 @ 10"
6. Check max. spacing of bars, \( s = 3(9) \leq 18 \quad \text{... s = 18" okay.} \)
7. One layer of reinforcement may be used because the wall thickness, \( h = 9" \leq 10" \).

31-2: Design a 20' long reinforced concrete bearing wall to support a slab bearing on the full thickness of the wall. The bottom and top of the wall are fixed connections. The wall is 22' high and the load from the slab, \( W_u = 2k/f \). \( f_c = 4\text{ksi}, \ f_y = 60\text{ksi} \).

1. \( b_{min} = (1/25)(22)(12) = 10.56" \quad \text{... b = 11".} \)
2. \( \Phi(.85f_c)A_1 = 0.65(.85)(4)(11)(12") = 291.72k > 2k/f(1') = 2k = P_u(\text{load on one-foot section}) \quad \text{... okay} \)
3. \( L_e = 12" \)
4. Check that axial load strength, \( \Phi P_n = 0.55 \Phi f_c A_g[1 − (kh/32b)^2] \)
   \( = .55(.65)(4)(11)(12)[1 − (.65(22)(12)/32(11))^2] = 143.9k > 6k/f(1') = 6k \quad \text{... okay.} \)
5. Select steel based on:
   - Vertical steel: \( A_s = .0144(11) = .158 \) use #4 @15"
   - Horizontal Steel: \( A_s = .024(11) = .264 \) use #4 @ 8.5"
6. Check that maximum spacing of bars, \( s = 3b = 3(11") = 33" \text{ or } s \leq 18" \quad \text{... s = 18".} \)
7. Two layers of reinforcement must be used because \( b = 11" > 10" \).
**31-3:** Design reinforcement for the shear wall shown in Figure 31.14. Use $f'_c = 4\text{ksi}$, $f_y = 60\text{ksi}$

1. Determine factored lateral loads. $F = 32k + 30k + 15k = 77k$
2. Determine lateral load $F_i = F_{\text{total}}(k_i/k_{\text{total}}) = 77k$
3. Assume the wall thickness, $b = 8"$. Assume placement of reinforcement on both faces.
4. $d = 0.8(L) = 0.8(16')(12"/f) = 153.6"$
   
   $\Phi V_n = 10\Phi(\sqrt{f'_c}bd) = 10(0.75)(\sqrt{4000})(8)(153.6)/1000k = 582.87k$
   
   $\Phi V_n = 582.87k > V_u = 77k$ ... okay

5. $\Phi V_c = 2\Phi(\sqrt{f'_c}bd) = 2(0.75)(\sqrt{4000})(8)(153.6)/1000 = 116.57k$
   
   $V_u = 77k < 116.57k = \Phi V_c$ ... shear reinforcement is NOT required.

11. $M_u = 15(40) + 30(28) + 32(16) = 1952k-ft = 23,424k-in$
12. Assume $\Phi = 0.9$ for flexure
13. $A_s = [0.85f'_c bd/fy][1 - \sqrt{1 - 2M_u/\Phi 0.85f'_c bd^2}] = [0.85(4)(153.6)/60][1 - \sqrt{1 - 2(23424)/[0.9(0.85)(4)(153.6)^2]}] = 2.88 \text{in}^2$
14. Check that $A_s \geq A_{\text{min}} = 3bd\sqrt{f'_c/fy} = 3(8)(153.6)(\sqrt{4000}/6000) = 3.89 \geq 200bd/fy = 200(8)(153.6)/6000 = 4.10$ ... therefore $A_{\text{min}} = 4.10 > 2.88$ ... use $A_s = 4.10$
15. From Table A.4.1 choose 4 - #10 with $A_s = 5.067 > 4.10\text{in}^2$.

Place 4 - #10 VEF (vertical each face) at each end.

**31-4:** Check the adequacy of the retaining wall in Figure 31.14 against overturning, sliding and sinking. Soil density = 80pcf, Concrete density = 150pcf, $\Phi = 23$ and Equivalent fluid pressure, $K_aW_e = 35\text{pcf}$. $P_a = 2000\text{psf}$

**Overturning:**

$M_o = 1.6H_a(h/3) = 1.6K_aW_eh^3/6 = 1.6(35)(10)^3/6 = 9333.33\#-f$

$K_p = 80\text{pcf}/35\text{pcf} = 2.286$

$M_{r1} = 1.6(2.286)(80\text{pcf})(8)^3/6 = 24,969.22\#-f$

$M_{r2} = 1.2(150)(1)(13)(3.5') = 8190\#-f$

$M_{r3} = 1.2(150)(12)(16/12)(6) = 17280\#-f$

$M_{r4} = 1.2(80)(8)(9')(8') = 55,296\#-f$

$M_{r5} = 1.2(80)(3)(4-16/12)(1.5) = 1152\#-f$

$M_r = 24969.22 + 8190 + 17280 + 55,296 + 1152 = 106,887.22\#-f$

Factor of Safety:

$M_r/M_o = 106,887.22/9333.33 = 11.45 > 1.5$ ... Retaining Wall will not overturn.

**Sliding:**

$K_p = 80\text{pcf}/35\text{pcf} = 2.286$

$W_1 = 1.2(150\text{pcf})(1')(13') = 2340\#-f$

$W_2 = 1.2(150\text{pcf})(12')(16/12) = 2880\#-f$

$W_3 = 1.2(80\text{pcf})(8')(9') = 6912\#-f$

$W_4 = 1.2(80\text{pcf})(3')(4'-16/12) = 768\#-f$

$\Sigma W = (2340 + 2880 + 6912 + 768)(1' \text{ thickness of wall}) = 12,900\#$

$F = 0.5(12900\#) = 6450\#$

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S = Ha – Hp = 1.6Kaweh^{2}/2 – 1.6Kpweh_{1}^{2}/2 = 1.6(35)(9)^{2}/2 – 1.6(2.286)(80)(4-16/12)^{2}/2 = 1227.62#

Factor of safety = F/S = 6450/1227.62 = 5.25 > 1.5 okay.

Mr = 106,887.22#-f and Mo = 9333.33#-f.

ΣW = 12,900#

X = (Mr – Mo)/ΣW = (106,887.22 – 9333.33)/12,900 = 7.56'

Centerline of the footing = 12'/2 = 6'

e = 7.56 – 6 = 1.56'

Soil pressure = ps = P/A ± Mc/I

ps\_toe = (ΣW/L)(1 + 6e/L) = (12900/12)(1+ 6(1.56/12) = 1913.5 \text{ @ the toe}

ps\_heel = (ΣW/L)(1 - 6e/L) = (12900/12)(1 – 6(1.1/7) = 236.5 \text{ @ the heel}

ps = 1913.5\text{psf} < pa = 2000\text{psf} \ldots \text{retaining wall is adequate against sinking.}

31-5: Design reinforcement for the retaining wall shown in Figure 31.14. Soil density = 80pcf, Concrete density = 150pcf, Φ = 23 and Equivalent fluid pressure, KaWe = 35pcf, Pa = 2000psf. f’c = 4ksi, fy = 60ksi

Shear in heel:

Factored Concrete weight: = 1.2(150pcf)(16/12')(1') = 240#/f↓

Factored Soil weight: = 1.2(80pcf)(1')(9') = 864#/f↓

ps = ps\_heal + (ps\_toe - ps\_heal)x/L = 236 + 139.75x

Soil bearing pressure changes from 236.5psf @ x = 0' to 236.5+139.75(8) = 1354.5psf @ x = 8', a difference of 139.75(8) = 1118psf.

W_1 = -864 – 240 + 236 = -868#/f

W_2 = 139.75x

Wx = -868 + 139.75x

Vx = W_1x + W_2x^2/2 = -868x + 69.88x^2

Vmax is at Wx = 0. X = 868/139.75 = 6.21'

Vu = -868(6.21) + 69.88(6.21)^2 = -2695.42k

Assume #8 bars

d = 16" – 2"cover(steel at top of footing) – .5 = 13.5"

ΦVc = Φ(2\sqrt{f'c})bd = .75(2)(\sqrt{4000})(12)(13.5) = 15,368.67# > Vu okay

ΦVc/2 = 15,368.67/2 = 7684.33# > 2695.42# = Vu \ldots \text{Stirrups are not required.}

Flexure in heel: Take the moment about x = 8'.

Wu = 8'(240+864) = 8832#↓ and c.g. is at x = 4'

P_1 = (1')(236.5psf)(8') = 1892#↑ and c.g. is at x = 4'

P_2 = (1')(139.75(8))(8')/2 = 4472#↑ and c.g. is at x = (8')/3 = 2.67'

Mu = -8832#(4') + 1892(4) + 4472#(2.67') = -15819.76#-f = 189837.12#-in

As = [.85fc’bd/fy][1 – \sqrt{[1 – 2Mu/Φ.85fc’bd^2]}

= [.85(4000)(12)(13.5)/60,000][1 – \sqrt{[1 – 2(189837.12)/(.9(.85)(4000)(12)(13.5)^3)]} = 0.264in^2

As\_min = greater of: 3bd/vfc’/fy = 3(12)(13.5)/4000/60,000 = 0.512 Or 200bd/fy = 0.54

As < As\_min = 0.54 therefore As = 0.54in^2

Use # 6 steel: A = 0.442in^2

Spacing ≥ .442(12)/.54 = 9.82"
Main Steel: #6@ 9"
Shrinkage steel: .0018(12") (16") = 0.35
18" ≥ Spacing ≥ .442(12)/.35 = 15.15"
Shrinkage Steel: #6 @ 15"

Shear in Toe:
Factored Concrete weight: = 1.2(150pcf)(1')(16/12) = 240#/f↓
Factored Soil weight: = 1.2(80pcf)(1')(4'-16/12) = 256#/f↓
Soil bearing pressure:
@ x = 9' w = 236.5+139.75(9) = 1494.25psf
@ x = 12' = 236.5 + 139.75(12) =
W₁ = -240 – 256 + 1494.25 = 998.25#/f↑
W₂ = 139.75x#/f↑
Wₓ = 998.25 + 139.75x
Vₓ = 998.25x + 69.88x²
V_max is at Wₓ = 3'
Vu = 998.25(3) + 69.88(3)² = 3623.67k
Assume #8 bars
d = 16" – 2"cover(steel at top of footing) – .5 = 13.5"
ΦVc = Φ(2√f’c)bd = .75(2)(√4000)(12)(13.5) = 15,368.67# > Vu okay
ΦVc/2 = 15,368.67/2 = 7684.33# > 3623.67# = Vu   … Stirrups are not required.

Flexure in toe: Take the moment about corner of stem and toe.
Wᵤ = 2'(240 + 256) = 496#↓ and c.g. is at x = 1.5'
P₁ = (1')(1494.25psf)(3') = 4482.75#↑ and c.g. is at x = 1.5'
P₂ = (1')(139.75(3'))(3')/2 = 628.88#↑ and c.g. is at x = 2(3')/3 = 2'
Mᵤ = 496#(1.5') – 4482.75#(1.5') – 628.88#(2') = -7237.89#-f = 86,854.68#-in
As = [.85fc'b'd/fy] [.85(4000)(12)(13.5)/(60,000)] [1 – √[1 – 2Mᵤ/.85fc'b'd²]] = 0.12in²
Asmin = 0.54; As < Asmin = 0.54 therefore As = 0.54. And the steel is the same as in the heel:
Main Steel: #6@ 9"
Shrinkage steel: #6 @ 15"

Shear in stem:
The horizontal force kₐWₑₕₑₕₑ varies from 0 @ y = 9’ to 35(9) = 315psf @ y = 0'
Hₐ = 1.6(315psf)(1')(9'/2) = 2268#,    Vu = Ha = 2268#
d = 12 – 2"cover(steel at outside of footing) – .5(#8 bars) = 9.5"
ΦVc = Φ(2√f’c)bd = .75(2)(√4000)(12)(9.5) = 10,814.99# > Vu = 2268# okay
ΦVc/2 = 10814.99/2 = 5407.49# > 2268#   ... no stirrups required.

Flexure in stem: Take moment at y = 0.
Mₑₕₑ = [(2268#/9'/3)] = 6804#-f = 81648#-in
As = [.85(4000)(12)(9.5)/(60,000)][1 – √[1 – 2(81648)/(.9(.85)(4000)(12)(9.5))²]] = 0.16in²
Asmin = 0.0018(12)(9.5) = 0.21 or 200(12)(9.5)/60000 = 0.38 therefore As = 0.38
Main Steel: Spacing ≥ .442(12)/.38 = 13.96"  USE #6 @ 13"
Shrinkage Steel: .0018(12") (12") = 0.26; s = .442(12)/.2620.4 = USE #6 @ 18"
Design a wall footing for an 8" concrete wall (t = 8''). DL = 6k/f, LL = 12k/f, f'c = 3ksi, 
fy = 60ksi, soil density = γs = 80pcf, allowable soil pressure = 4000psf. The bottom of the footing 
must be 4.0' below grade.

1. Compute factored loads: (1.2(6) + 1.6(12))(1') = 26.4k = Pu 
   Unfactored loads = P = (6 + 12)(1') = 18k
2. Assume footing thickness: h = 18''
3. \( w_{ftg} = 0.15\text{kcf}(1.5') = 0.225\text{ksf} \)
4. \( w_s = 80\text{pcf}(4' – 1.5') = 200\text{psf} = 0.2\text{ksf} \)
5. Net allowable soil pressure = \( p_{net} = p_s - w_{ftg} - w_s = 4.0\text{ksf} – .225\text{ksf} – .2\text{ksf} = 3.575 \text{ksf} \)
6. Maximum allowable soil pressure = \( p_{max} = (Pu/P)(p_{net}) = (26.4/18)(3.575\text{ksf}) = 5.24\text{ksf} \)
7. Required footing width = \( L_1 = Pu/p_{max} = 26.4/5.24 = 5.04' \), round up to 5'–4'' = 5.33' = 64''
8. Recalculate factored soil pressure: \( = pu = Pu/L_1 = 26.4k/f/5.33' = 4.953\text{ksf} < 5.24\text{ksf} \)  … okay
9. Find effective depth assuming #8 bars: 
   \( d = 18'' – 3''\text{cover} – .5'' = 14.5'' \)
10. Shear reinforcement is not required in footings if \( \Phi V_c > V_u \). Since the footing width is 5.33' and wall width is 8''/12''/f = 0.67', the length of the footing on either side = (5.33 – .67)/2 = 2.33'. 
   \( d = 14.5/12 = 1.208'' \)

\( V_u = (2.33 – 1.208')(1')\)\(5.24\text{ksf}) = 5.88k \)
\( \Phi V_c = .75(2)\sqrt{(f'c)d(12''}/1000\#/	ext{k} = 14.3k \)
Since \( \Phi V_c = 14.3 > 5.88 = V_u \) … No shear reinforcement necessary.
11. \( M_{max} \) is at 1/4 of the wall thickness into the wall. Wall thickness = 8'' \( \ldots M_{max} \) is 2'' into the wall. 
    Moment arm = 2''/12''/f + 2.33' = 2.5' 
    \( Mu = 5.24\text{ksf}(2.5\text{ft})^2/2 = 16.375\text{k-f} = 196,500\#\text{-in} \)
12. \( As = 0.85\text{f'}cbd/fy[1.2\sqrt{(f'c)d(12'')}/1000\#/	ext{k} = 14.3k \)
\( = [.85(3000)(12)(14.5)]/60000[1 - \sqrt{1 - 2(196,500\#\text{-in})/.9(.85(3000)(12)(14.5''))^2} \)
   \( = 0.255\text{in}^2/\text{ft of wall} \)
As min = bd(3\text{f'}c)/fy ≥ 200bd/fy for beams \( 3\text{f'}c = 164.32 < 200 \) … use 200. 
   As min = 200\(\)\(12'')(14.5'')/60000 = 0.58 for beams and
   As min = .0018\(\)\(12'')(18') = 0.389 for slabs
Use larger of the three values: \( As = 0.58\text{in}^2/	ext{f} \)
#6: \( A = 0.442, \) spacing = 12''(0.442/0.58) = 9.14'' round down to 9.0''
Transverse steel: USE #6 @ 9''o.c. \( As = 0.589\text{in}^2 \)
13. Check development length of the Transverse bars: 
    \( Kd = 3fy/40\text{f'}c = 3(60000)/[40(\sqrt{3000})] = 82.16 \)
    \( \psi t = 1.0, \psi e = 1.0, \psi s = 0.8 \) \( #6 \) or smaller bars, \( \lambda = 1.0 \)
    \( \psi_{te} = 1.0 < 1.7 \) okay
    \( c_b = \text{smaller of cover or half spacing: cover = 3'\)'', spacing = 9/2 = 4.5'' \ldots c_b = 3'' \)
    \( K_{tr} = 0; \) \( (c_b + K_{tr})/db = 3.0/0.75 = 4.0'' > 2.5'' \ldots \) Use 2.5''
    \( Ld = (Kd/\lambda)(\psi_{te} \psi_{es})(db)/[(cb + K_{tr})/db] = 82.8(8)/.75/2.5 = 19.728 \)
    You may use Ker factor = \( As_{req'}/As_{used} = .58/.589 = .985 \)
    \( Ld = 19.728(.985) = 19.432'' \)
    \( Ld \text{ provided} = \text{critical length for moment – 3'' cover = 2.33'(12''/f) – 3'' = 24.96'' > 19.06'' \ldots \) okay
14. Longitudinal steel: \( As_{min} = .0018bh = .0018(5.33')(12''/f)(18') = 2.07\text{in}^2 \)
    USE 5 - #6 bars spaced equally
Design an individual column where: DL = 200k, LL = 500k, Allowable Soil Pressure = 3ksf, $f'_{c,\text{col}} = 4\text{ksi}$, $f'_{c,\text{ftg}} = 3\text{ksi}$, soil density = 80pcf, 28"X28" column, bottom of footing is 3' below grade, supporting interior column.

1. Assume footing thickness $h = 24''$
2. Find net allowable soil pressure $p_{\text{net}} = 3\text{ksf} - .15(24/12) - .08(12/12) = 2.62\text{ksf}$
3. Required Footing Area $A_{\text{REQ}} = (200 + 500)/2.62 = 267.18\text{ft}^2$
   Round up: Footing size = 16.5' by 16.5' = 272.25\text{ft}^2
4. Find Factored soil pressure $p_{\text{u}} = p_{\text{u}}/A = (1.2(200) + 1.6(500))/272.25 = 3.82\text{ksf}$
5. Calculate $d = h - 3'' - d_0 = 24 - 3 - 1 = 20''$ (assuming #8bars)
6. One way shear – beam shear:
   \[ G = (16.5(12) - 28)/2 - 20 = 65'' \]
   \[ V_u = p_u L_2 G = 3.82\text{ksf}(16.5')(65/12) = 341.41\text{k} \]
   \[ V_c = 2\sqrt{f'cbd} = 2\sqrt{3000\text{psi}(198\text{in})(20\text{in})/1000\#/k} = 433.80\text{k} \]
   \[ 341.41\text{k} < .75(433.80\text{k}) = 325.35\text{k} \] ... increase depth of footing

1A. Assume footing thickness $h = 33''$
2A. Find net allowable soil pressure $p_{\text{net}} = 3\text{ksf} - .15(33/12) - .08(4-33/12) = 2.49\text{ksf}$
3A. Required Footing Area $A_{\text{REQ}} = (200 + 500)/2.49 = 281.12\text{ft}^2$
   Round up: Footing size = 17' by 17' = 289\text{ft}^2
4A. Find Factored soil pressure $p_{\text{u}} = p_{\text{u}}/A = (1.2(200) + 1.6(500))/289 = 3.60\text{ksf}$
5A. Calculate $d = h - 3'' - d_0 = 33 - 3 - 1 = 29''$ (assuming #8bars)
6A. One way shear – beam shear:
   \[ G = (17(12) - 28)/2 - 29 = 59'' \]
   \[ V_u = p_u L_2 G = 3.6\text{ksf}(17')(59/12) = 300.9\text{k} \]
   \[ V_c = 2\sqrt{f'cbd} = 2\sqrt{3000\text{psi}(204\text{in})(29\text{in})/1000\#/k} = 648.07\text{k} \]
   \[ 300.9\text{k} < .75(648.07\text{k}) = 486.05\text{k} \] ... okay, no stirrups required.

Two way shear – punching shear
\[ B = 28 + 29 = 57'' \]
\[ b_o = 48 = 228''; \ a = 40 \text{ for interior column} \]
\[ V_u = p_u(W^2 - B^2) = 3.6(17^2 - 4.75^2) = 959.18\text{k} \]
\[ V_c = \text{smallest of:} \]
   \[ V_c = (2 + 4/\beta_c)\sqrt{f'cbd} = 6\sqrt{3000(228)(29)/1000} = 2172.92\text{k} \]
   Or
   \[ V_c = (a/sd/b_o + 2)\sqrt{f'cbd} = (40(29)/228 + 2)\sqrt{3000(228)(29)/1000} = 2566.84\text{k} \]
   Or
   \[ V_c = 4\lambda\sqrt{f'cbd} = 4(1)\sqrt{3000(228)(29)/1000} = 1448.62\text{k} \]
\[ V_u = 959.18 < \Phi V_c = .75(1448.62) = 1086.46\text{k} \] ... OKAY!

... the footing is adequate for punching shear.
8. Calculate Moment:
\[ F = \frac{(L_1 - t)/2}{2} = \frac{(17 - 28/12)/2}{2} = 7.33' \]
\[ Mu = \frac{pL_2}{(F/2)} = \frac{3.6(17)(7.33)/2}{2} = 16.44.1k-f = 19,729.25k-in \]

9. Find area s steel required: \((b = L_1)\)
\[ As = 0.85f'cbd/fy[1 - \sqrt[1 - 2Mu/\Phi(0.85f'cbd^2)] in^2 \]
\[ = [0.85(3000)(17')(12''/f)(29'')/60000] [1 - \sqrt[1 - 2(19729.25k-in)(1000#/k)/0.85(3000psi)(17') \]
\[ (12''/f)(29'')] = 12.93in^2 \]
\[ As_{min} = bd(3\sqrt{f'}c)/fy = 200bd/fy = 200(17')(12)(29)/60000 = 19.72in^2 \]
\[ As_{min} = 0.0018bh = 0.0018(17')(12'')/f(29'') = 10.65in^2 \]
USE larger of the three values: \(As = 19.72in^2\): USE 26-#8 evenly spaced

10. Check development length of steel
\[ Kd = \frac{3fy/40\sqrt{f'c}}{3(60000)/[40(\sqrt{3000})]} = 82.16 \]
\[ \psi t = 1.0, \psi e = 1.0, \psi s = 1 \quad (b = 8 \text{ bars}) \quad \lambda = 1.0 \quad \psi t\psi e = 1.0 < 1.7 \quad \text{okay} \]
\[ c_b = \text{smaller of cover ( = 3'') or half spacing ( = (17''(12''/f) - 6'' - 1'')/25 = 7.88'' spacing) ...} \]
\[ c_b = 3''; Ktr = 0 \]
\[ (c_b + Ktr)/d_b = 3.0/1 = 3'' > 2.5'' \quad \text{Use 2.5''} \]
\[ L_d = (Kd/\lambda)(\psi t\psi e\psi s)/[cb + Kt]/db \quad [As \text{ req'd}/As \text{ used}] = [82.8(1)(1)/2.5][19.72/20.41] \]
\[ = 32.00'' \]
\[ L_d \text{ provided} = \text{critical length for moment} - 3'' \text{ cover} = 7.33(12) - 3 = 84.96'' > 32.00'' \quad \text{okay} \]

11. Check bearing strength at base of column for column and for footing are > Pu.
\[ A_1 = t_2 = 28^2 = 784in^2 \]
\[ A_2 = L_1^2 = 204^2 = 41616in^2 \]
*Use \(f'c = 4ksi\) for column ** use \(f'c = 3ksi\) for footing

Col. Bearing strength = \(\Phi(.85f'cA_1) = .65(.85)(4ksi)(784) = 1732.64 > 1040K = Pu \quad \text{okay} \)
Footing Bearing Strength = \(\Phi(.85f'cA_1)\sqrt{A_2/A_1} < \Phi(.85f'cA_1)(2) \]
\[ \sqrt{A_2/A_1} = 7.29 > 2 \quad \text{use 2} \]
\[ \Phi(.85f'cA_1)(2) = 65(.85)(3ksi**)(784)(2) = 2598.96 > 1040k = Pu \quad \text{okay} \]

12. Calculate dowel Area = \(A_{sd} = .005A_1 = .005(784) = 3.92in^2 \quad \text{... use 4 - # 9, As = 4.0in^2} \)
Check development length:
\[ L_{dc} = (0.02fy/\lambda\sqrt{f'c})(db)(\text{Required Asd}/\text{provided Asd}) \geq 0.0003fyd_b \]
\[ L_{dc} = (0.02(60000)/(1)\sqrt{3000})(1.128)(3.92)/4 = 24.22'' > 0.0003fyd_b = 20.3'' \]
USE 4 - #9 X 24.5'' long.
32-3: Design a rectangular combined footing for the two columns shown below. The allowable soil pressure = 3.5ksf, f’c = 3ksi and fy = 60ksi. Soil density = 90pcf. Column A is 18” X 18” and carries a dead load of 100k and live load of 300k. Column B is 20” X 20” and carries a dead load of 100k and a live load of 500k.

1. R = resultant of the column loads. NOTE: Do not factor loads
   \[100 + 300 + 100 + 500 = 1000k = R\]
   X = the location of the resultant, R
   \[1000X = (100 + 300)(0) + (100 + 500)(20') = 12000k-f\]
   \[X = 12000/1000 = 12'\]

2. Find the length of footing:
   maximum distance to left = 12 + 4’ = 16’
   maximum footing length = 16’(2) = 32’ = 384”
   L = 32.0ft.

3. Find footing width:
   assume h = 24in
   net soil pressure: \[p_{net} = 3.5ksf - 2(0.15) - (0.08ksf)(1') = 3.12ksf\]
   \[A = R/p_{net} = 1000/3.12 = 320.51sf\]
   \[b = A/L = 320.51/32.0 =10.02'\]
   round up to \(W = 10.25' = 123''\)

4. Draw shear diagram & find \(M_u\) in longitudinal direction: USE FACTORED LOADS:
   Column A: \(P_L = 1.2(100) + 1.6(300) = 600k\)
   Column B: \(P_R = 1.2(100) + 1.6(500) = 920k\)
   Soil weight and beam weight can be ignored because their effect is offset by an equivalent uniform soil pressure.
   The uniform reaction in response to the column loadings = \((600 + 920)/32.0f = 47.5k/f\)

   NOTE: for a more accurate moment diagram, create uniform loads over the column widths.
Mu = -1389.56k-f = 16674.72k-in
5. let a = 0.2d,
d = \sqrt{\left(\frac{Mu}{f \cdot 1.153\Phi f'cb}\right)}
= \sqrt{\left(\frac{16674.72}{0.153(9)(3ksi)(123')}\right)} = 18.12''
6. Find depth for one way and punching shear: Consider Column A to have
L1 = 4' + 8.63' = 12.63' and Column B to have L1 = 32' - 12.63' = 19.37'

One way shear – beam shear
Column A:
Vu = 410k - (9'' + d'')(47.5k/f)/(12''/f)
\quad = 374.38 - 3.958d
ΦVc = .75(2)√3000psi(123)d/1000#/k = 10.11d
d = 374.38/(10.11 + 3.958) = 26.61''
Column B: Vu = 540 - (10 + d)(47.5)/12
\quad = 500.42 - 3.958d
ΦVc = .75(2)√3000psi(123)d/1000#/k = 10.11d
\quad d = 500.42/(10.11 + 3.958) = 35.571''
USE d = 36'', h = 39''

Check Two way shear – punching shear

Column A: (edge column)
A1 = 12.63'(10.25') = 129.46f²
A2 = (18'' + 36'')² / (144in²/f²) = 20.25f²
\sqrt{129.46/20.25} = 2.53 > 2.0 \quad \beta_c = 2
Vu = pu(A1 - A2) = ((47.5k/f)/10.25')(129.46 - 20.25) = 506.1k
bo = 4(18 + 36) = 216''
Vc = smallest of:
\quad Vc = (2 + 4/\beta_c)\sqrt{f'cb_o}d = (4)(\sqrt{3000})(216)(36)/1000 = 1703.64k
\quad Vc = (a_d/b_o + 2) \sqrt{f'cb_o}d = (30(36)/216 + 2)(\sqrt{3000})(216)(36)/1000 = 2981.37k
\quad Vc = 4\sqrt{f'cb_o}d = (4)(\sqrt{3000})(216)(36)/1000 = 1703.64k
ΦVc = .75(1703.64) = 1277.73k > 506.1k... okay

Column B: (interior column)
A1 = 19.37'(10.25') = 198.54f²
A2 = (20'' + 36'')² / (144in²/f²) = 21.78f²
\sqrt{198.54/21.78} = 3.02 > 2.0 \quad \beta_c = 2
Vu = pu(A1 - A2) = ((47.5k/f)/10.25')(198.54 - 21.78) = 819.13k
bo = 4(20 + 36) = 224''
Vc = smallest of:
\quad Vc = (2 + 4/\beta_c)\sqrt{f'cb_o}d = (4)(\sqrt{3000})(224)(36)/1000 = 1766.73k
\quad Vc = (a_d/b_o + 2) \sqrt{f'cb_o}d = (40(36)/224 + 2)(\sqrt{3000})(224)(36)/1000 = 3722.75k
\quad Vc = 4\sqrt{f'cb_o}d = (4)(\sqrt{3000})(224)(36)/1000 = 1766.73k
ΦVc = .75(1766.73) = 1325.05 > 819.13k... okay
7. Compute flexural steel for positive moment:

Column A: \( \mu = 380k-f = 4,560,000\text{#-in} \)

\[
\begin{align*}
As &= 0.85f'cbd/fy[1 - \sqrt{1 - 2\mu/\Phi(0.85f'cbd^2)}] \\
&= [0.85(3000)(123)(36)/60000][1 - \sqrt{1 - 2(4,560,000)/(0.9)(0.85(3000)(123)(36)^2)}] = 2.36\text{in}^2 \\
\end{align*}
\]

USE 2 - #11

Column B: \( \mu = 1680.75k-f = 20,169,000\text{#-in} \)

\[
\begin{align*}
As &= 0.85f'cbd/fy[1 - \sqrt{1 - 2\mu/\Phi(0.85f'cbd^2)}] \\
&= [0.85(3000)(123)(36)/60000][1 - \sqrt{1 - 2(20,169,000)/(0.9)(0.85(3000)(123)(36)^2)}] = 10.68\text{in}^2 \\
\end{align*}
\]

USE 7 - #11

8. Compute flexural steel for negative moment

\( \mu = 1389.56k-f = 16,674,720\text{#-in} \)

\[
\begin{align*}
As &= 0.85f'cbd/fy[1 - \sqrt{1 - 2\mu/\Phi(0.85f'cbd^2)}] \\
&= [0.85(3000)(123)(36)/60000][1 - \sqrt{1 - 2(16,674,720)/(0.9)(0.85(3000)(123)(36)^2)}] = 8.78\text{in}^2 \\
\end{align*}
\]

USE 6 - #11

Compute transverse steel:

\[
\begin{align*}
F &= (10.25' - 18/12)/2 = 4.375' \\
\mu &= [(4.375)^2/2](1')[47.5k/f/10.25'] = 44.35k-f = 532,202.74\text{#-in} \\
As &= 0.85f'cbd/fy[1 - \sqrt{1 - 2\mu/\Phi(0.85f'cbd^2)}] \\
&= [0.85(3000)(12)(36)/60000][1 - \sqrt{1 - 2(532,202.74)/(0.9)(0.85(3000)(12)(36)^2)}] = 0.275\text{in}^2/f \\
As_{\min} &= 0.0033(12''/f)(36'') = 1.43\text{in}^2/f \\
As_{\min} &= 0.0018(12''/f)(39'') = 0.84\text{in}^2/f \\
\end{align*}
\]

USE #11 @ 13'' o.c.
**33-1:** Design a 40' long 14"X30" beam with f'c = 4000psi, a DL of .1k/f and a LL of .8k/f using 2 tendons with a sag of 12".

a) Determine pre-stressing force required if 2 parabolic tendons with a sag of 12" are used.

Beam weight = w = .15pcf(14/12)(30/12) = 0.438k/f
Factored load = 1.2(0.438 + .1) + 1.6(.8) = 1.925k/f
Moment = Mu = 1.925k/f(40)^2/8 = 385k-f = 4,620,000#-in
Pre-stressing force = P = Mu/sag = 4620000/[12"\times2\text{tendons}] = 192,500#

b) Check equivalent stress block depth, a:

\[ f_{\text{app}} = 0.85f'c - P/bh = 0.85(4000) - 192,500/(14\times30) = 2941.67\text{psi} \]
\[ a = d - \sqrt{d^2 - 2Mu/\Phi bf_{\text{app}}} = 27 - \sqrt{27^2 - 2(4620000)/(0.9)(2941.67)} = 5.09" \]
\[ c = a/0.85 = 6.80" < h/2 = 30/2 = 15" \text{ ... okay.} \]

**33-2:** Design a 48' long, 12" deep slab with a a LL of .1ksf using tendons every 6".

a) Determine the pre-stressing force required for parabolic tendons with a sag of 3" @ 6" o.c.

d = 12-1.125 = 10.875"
Slab weight = w = .15pcf(12/12) = 0.15k/f
Factored load for 12" swath = [1.2(0.15) + 1.6(.1)]1' = 0.34 k/f
Moment = Mu = .34k/f(48)^2/8 = 97.92k-f = 1,175,040#-in
Pre-stressing force = P = Mu/sag = 1,175,040k-in/[3\times2] = 195,840#

b) Check equivalent stress block depth, a:

\[ f_{\text{app}} = 0.85f'c - P/bh = 0.85(4000) - 195,840/(12\times12) = 2042.5\text{psi} \]
\[ a = d - \sqrt{d^2 - 2Mu/\Phi bf_{\text{app}}} = 10.875 - \sqrt{10.875^2 - 2(1175040)/(0.9)(2042.5)} = 7.45" \]
\[ c = 7.45"/0.85 = 8.76" > 12/2 = 6" \text{ ... decrease prestressing force.} \]
Let P = 100,000; \ f_{\text{app}} = 0.85(4000) - 100000/144 = 2705.56
\[ a = 10.875 - \sqrt{10.875^2 - 2(1175040)/(0.9)(2705.56)} = 4.72"; \ c = 4.72/0.85 = 5.56" < 6" \text{ okay.} \]
34-1: Design reinforcement for a 16" thick masonry wall filled and reinforced with vertical 60ksi rebar. There is a uniform load of 4800#/f centered on the wall and a lateral force of 20psf on the surface of the wall. The wall is 30' high and the filled weight = 145pcf. F'm = 3000psi

1. Determine Mu for one foot swath of wall: 
   \[ Mu = 20(30)^2/8 = 2250#-f = 27000#-in. \]

2. \[ As = (.8f'mbd/fy)(1 - \sqrt[1 - (2Mu/.8f'mbd^2)]) \]
   \[ = (.8(3000)(12)(12)/(60000))(1 - \sqrt[1 - (2(27000)/.8(3000)(12)(12^2)]) \]
   \[ = 0.037in^2/f = \#5 @ 18"o.c. \]

3. \[ fb = M/S = 27000#-in/[(12")^2/6] = 52.73psi \]

4. \[ Fb = 0.45f'm = .45(3000) = 1350psi \]

5. \[ fa = P/A = 145pcf(30')/144 + 4800#/f/[12"/f(16'')] = 55.21psi \]

6. \[ Fa = .8f'm = .8(3000) = 2400psi \]

7. \[ fa/Fa + fb/Fb = 52.73/1350 + 55.21/2400 = 0.062 < 1.0 \ldots \text{okay}. \]

USE \#5 @ 18"o.c.