

Trigonometric Identities

Sum and Difference Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Half-Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos x}{\sin x}$$

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$$

Double-Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

Product-to-Sum Formulas

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

$$\sin x - \sin y = 2 \sin \left(\frac{x-y}{2} \right) \cos \left(\frac{x+y}{2} \right)$$

$$\cos x + \cos y = 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

$$\cos x - \cos y = -2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Suppose you are given two sides, a, b and the angle A opposite the side A . The height of the triangle is $h = b \sin A$. Then

1. If $a < h$, then a is too short to form a triangle, so there is no solution.
2. If $a = h$, then there is one triangle.
3. If $a > h$ and $a < b$, then there are two distinct triangles.
4. If $a \geq b$, then there is one triangle.

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$